GEOPHYSICAL TRANSACTIONS 1997
Vol. 40. No. 3-4. pp. 155-174

VSP inversion — A new method using edge detection

Zoltán WÉBER*

A novel approach for obtaining interval velocity estimates from vertical seismic profiles (VSPs) is described. The edge detection technique, well known in image processing, can supply large amount of gradient data from the VSP image. Statistical evaluation of several gradient values obtained for the same depth interval yields reliable velocity estimates and error limits.

The preprocessing sequence contains only wavelet filtering and wavefield separation. True amplitude processing and definition of initial velocity model are not necessary. The method can handle offset (OVSP) data as well. In order to obtain the interval velocities from OVSP data, the gradient values are multiplied by a correction factor, which is determined by ray trace modelling.

Several tests on synthetic and real data have proved that the method is efficient and relatively independent of random noise and model errors. The procedure can be carried out economically on personal computers.

Keywords: VSP inversion, edge detection, most frequent value

1. Introduction

Vertical seismic profiling (VSP) is becoming increasingly recognized as a valuable technique in seismic exploration [GALPERIN 1974, HARDAGE 1985, BALCH and LEE 1984]. Perhaps the most vital information inferred from VSP is the seismic velocity. A major processing task is to determine the velocity-depth function, which becomes especially difficult when the source is offset

* formerly: Geophysical Research Group of the Hung. Acad. Sci. Department of Geophysics, Eötvös University H-1083 Budapest, Ludovika tér 2., Hungary
presently: Seismological Observatory of the Hung. Acad. Sci., H-1112 Budapest, Meredek u. 18., Hungary

Manuscript recevied: 21 August, 1995

some distance from the wellhead. Source offset, however, is often necessary to avoid or reduce the generation of tube waves and to sample the subsurface away from the borehole in order to obtain two-dimensional information.

Several methods are currently in use to compute interval velocities from offset VSP (OVSP) data. The simplest method assumes vertical propagation and computes an apparent velocity $\Delta z/\Delta t$, where Δz is the difference in geophone depths and Δt is the corresponding difference in the arrival times. Better approximation is achieved by assuming slanted straight rays [LASH 1980]. The next step towards more accurate velocities is based on a flat layered model using Snell's law to predict the direction of ray propagation [GRANT and WEST 1965].

An even more sophisticated method is proposed by STEWART [1984], who solves the problem using a generalized linear inversion (GLI) technique. He assumes an initial velocity structure and perturbs it iteratively until the computed traveltimes match the observed ones in a least squares sense. At each iteration a ray tracing technique is used to compute traveltimes. LINES et al. [1984] use a different but related method to determine layer dips from VSP arrival times and sonic log velocities. PUJOL et al. [1985] propose a layer stripping technique and a generalized inversion somewhat similar to that of STEWART [1984].

The methods described so far have inherent limitations. The first two are reasonable approximations only for small offsets. Since traveltimes cannot be determined with sufficient accuracy neither interactively nor automatically, all methods based on arrival times are prone to error [STEWART 1984, BALCH and LEE 1984].

Specifically for zero-offset VSP inversion GRIVELET [1985], MACE and LAILLY [1986], URSIN [1986] and others have proposed GLI methods, which use the whole VSP wavefield. Since the problem is nonlinear, the initial estimate of the acoustic impedance function must be close enough to the real one in order to get the global, and not the local, minimum of the objective function. Because the GLI method utilizes the dynamic features of the VSP data instead of the velocity function, it is the acoustic impedance function which can be estimated and, on the other hand, the amplitudes of the VSP data have to be correctly reconstructed during data processing. Another disadvantage of the GLI technique follows from the huge amount of computations. The calculation of realistic synthetic VSPs and the solution of the large sets of linear equations require extremely fast computers.

The interval velocity estimation technique described in this paper uses a much simpler approach. The gradient of the traveltime curve of the first arrivals is estimated at each geophone depth by means of edge detection, a standard procedure of image processing [e.g. ROSENFELD and KAK 1982]. In order to obtain the interval velocities from OVSP data, the gradient values are multiplied by a correction factor, which is determined by ray trace modelling, assuming a horizontally layered earth model. The preprocessing sequence of the VSP data contains only wavelet filtering and wavefield separation. By virtue of the statistical processing of the gradient data calculated by edge detection, the error of the estimation can be given in addition to the estimated velocities. The definition of an initial velocity model is not necessary.

Several tests on synthetic and real data have proved that the method is efficient and relatively insensitive to random noise and model errors. The procedure needs so little computer time that it can be realized economically even on personal computers.

2. The gradient method based on edge detection

The series of first breaks on a VSP defines the traveltime curve of the first arrivals. As is well known, the gradient of this traveltime curve (i.e. $\Delta z / \Delta t$) is closely connected with the wave velocity: in the case of zero-offset VSP and horizontal layering the gradient is equal to the interval velocity.

The gradient of the traveltime curve as a function of depth can be estimated by edge detection. On the VSP, which is considered a two-dimensional image, the gray levels change rapidly in the vicinity of the first arrivals because of the large amplitudes. The dominant direction of these rapid changes should be found at each geophone level. The Sobel operator, which is a 3_3 point two-dimensional edge detection operator, determines edge directions at each data point of the VSP. At a given geophone depth the most probable value of the gradient, characterizing the traveltime curve of the first arrival at this depth, can be estimated by a statistical method using several gradient values in an appropriately chosen time gate around the first break. The width of this time gate is usually between about 100 and 200 ms. Such a time gate includes 50-100 gradient values if the sampling interval is 2 ms. The statistical processing of the large number of gradient data involves that in addition to the estimated velocity values the error of the estimation can also be given. In this paper the most frequent value procedure is used because of its robustness and resistance [STEINER 1988].

Experience gained during the tests with the gradient method suggests that edge detection works more effectively on the instantaneous phase section calculated from the VSP than on the original VSP data. (For the computation of instantaneous phase see e.g. TANER et al. [1979]). The reason for this is that the instantaneous phase section contains sharper 'jumps' than the original section. As a consequence, the gradient values obtained from the instantaneous phase are more accurate than those calculated from the original VSP data.

3. Application of the gradient method to OVSP data

In the case of zero-offset VSP and horizontally layered media, gradients provide good estimates of the velocities, but this is not true for gradients computed from OVSP data.

In order to find the relation between the gradient values and the velocities in the latter case, let us assume that the traveltimes of the first arrivals to the depths z_i and z_{i+1} are t_i and t_{i+1}, respectively, and the wave arriving to the geophone at depth z_{i+1} travels the path length Δs_i in the time interval $\Delta t_i = t_{i+1} - t_i$ (*Fig. 1*). Because, by time t_i, this wave is already in the ith layer, the velocity in this depth interval is $\Delta s_i / \Delta t_i$ to a good approximation. (Here the depth interval $[z_i, z_{i+1}]$ is considered as the ith layer.) Moreover, let us assume that the velocity is equal to the gradient $\Delta z_i / \Delta t_i$ multiplied by a correction factor C_i, i.e.

$$\frac{\Delta s_i}{\Delta t_i} = C_i \frac{\Delta z_i}{\Delta t_i} ,$$

where Δz_i denotes the thickness of the ith layer. It is obvious that the correction factor is

$$C_i = \frac{\Delta s_i}{\Delta z_i}$$

In the case of zero-offset VSP, Δs_i is equal to Δz_i, i.e. the correction factor C_i is equal to 1. Now the question is how to determine the correction factor C_i if the offset is not zero.

Let us assume that the interval velocities are known to depth z_i, and the calculated traveltime to this depth is t_i. The traveltime t_{i+1} to depth z_{i+1} can also be calculated if an arbitrary velocity value v_i is assumed in the ith

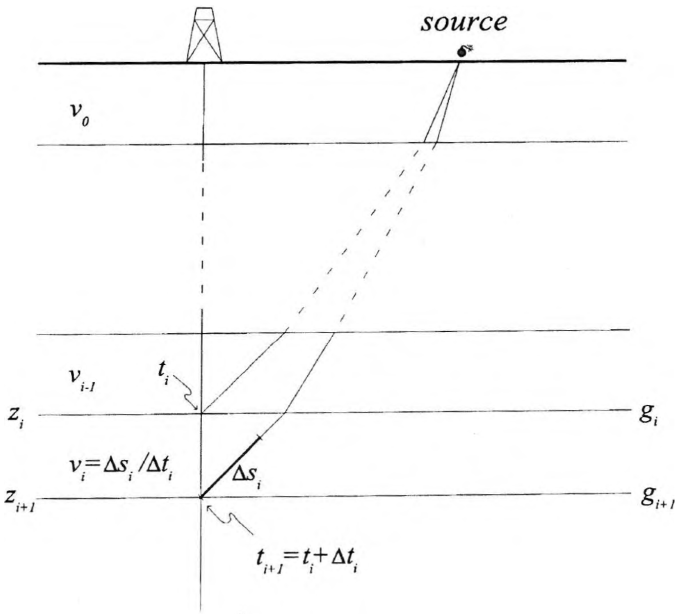


Fig. 1. Offset VSP geometry for velocity estimation in a horizontally layered earth model using the gradient data g_i

1. ábra. Offszetes VSP mérés geometriája horizontális rétegzettség mellett a g_i irányadatok alapján végzett sebességbecsléshez

layer. In the time interval $[t_i,\ t_{i+1}(v_i)]$ the wave travels the path length $\Delta s_i = v_i\ [t_{i+1}(v_i)-t_i]$, i.e. the correction factor is

$$C_i = \frac{v_i\left[t_{i+1}(v_i)-t_i\right]}{\Delta z_i}\,. \tag{1}$$

Velocity v_i has been chosen correctly if it is equal to gradient g_i of the traveltime curve of the first arrival multiplied by the correction factor C_i:

$$C_i g_i = v_i\,. \tag{2}$$

Substituting Eq. (1) into Eq. (2):

$$\frac{t_{i+1}(v_i)-t_i}{\Delta z_i}\, g_i = 1 \tag{3}$$

If Eq. (3) is not satisfied, velocity v_i should be modified until the equality becomes valid within an acceptable tolerance.

In order to calculate the raypath and the traveltime of the first arrival, a system of nonlinear equations should be solved. The Newton-Raphson method has been used here (see Appendix). This iterative method requires an initial approximation. The iteration is convergent only if this initial approximation is close enough to the actual solution. In our problem the simplest way to define an initial guess is to assume a straight raypath between the source and the geophone. However, this approximation is close enough to the actual solution only if the unknown velocity v_i is close enough to the velocity in the $(i$-$1)$st layer.

Based on the foregoing discussion, the following procedure is proposed for finding the correct velocity v_i. In the first step let the unknown velocity v_i be equal to the (known) velocity v_{i-1}. Under this condition the traveltime t_{i+1} can be calculated. If the left hand side of Eq. (3) is greater than 1, then t_{i+1} is too large, i.e. velocity v_i should be increased, otherwise v_i should be decreased. According to the result of the first step, v_i should be altered by a preset velocity increment Δv. The raypath calculated in the previous step can be used as an initial approximation in the calculation of the new raypath and traveltime t_{i+1} of the first arrival. If the left hand side of Eq. (3) is still not close enough to 1, then velocity v_i should again be altered by Δv. This procedure is continued until Eq.(3) holds to an acceptable tolerance.

4. Synthetic examples

The features of the inversion method described in the previous sections have been studied on synthetic data. By means of the examples we examine how the method is influenced by random noise and model errors.

The velocity and density functions used in synthetic zero-offset VSP calculation consist of 1000 layers. Each layer is 2 metres thick. The calculated synthetic VSP consists of 139 traces, the distance between the equally spaced geophones (traces) is 10 metres, the sampling rate is 2 ms. A Klauder wavelet with peak frequency of 30 Hz is assumed. The forward method used here to compute the synthetic VSP takes into consideration the effects of absorption and dispersion [Ganley 1981].

The downgoing wavefield used in the velocity estimation and the corresponding instantaneous phase section are shown in *Fig. 2*. Since the instantaneous phase section contains much sharper 'jumps' than the original VSP, the gradient data were calculated from the instantaneous phase section. For the velocity estimation 200 ms wide time gates were used around the first break at each geophone level. The final results are shown in *Fig. 3*. (The

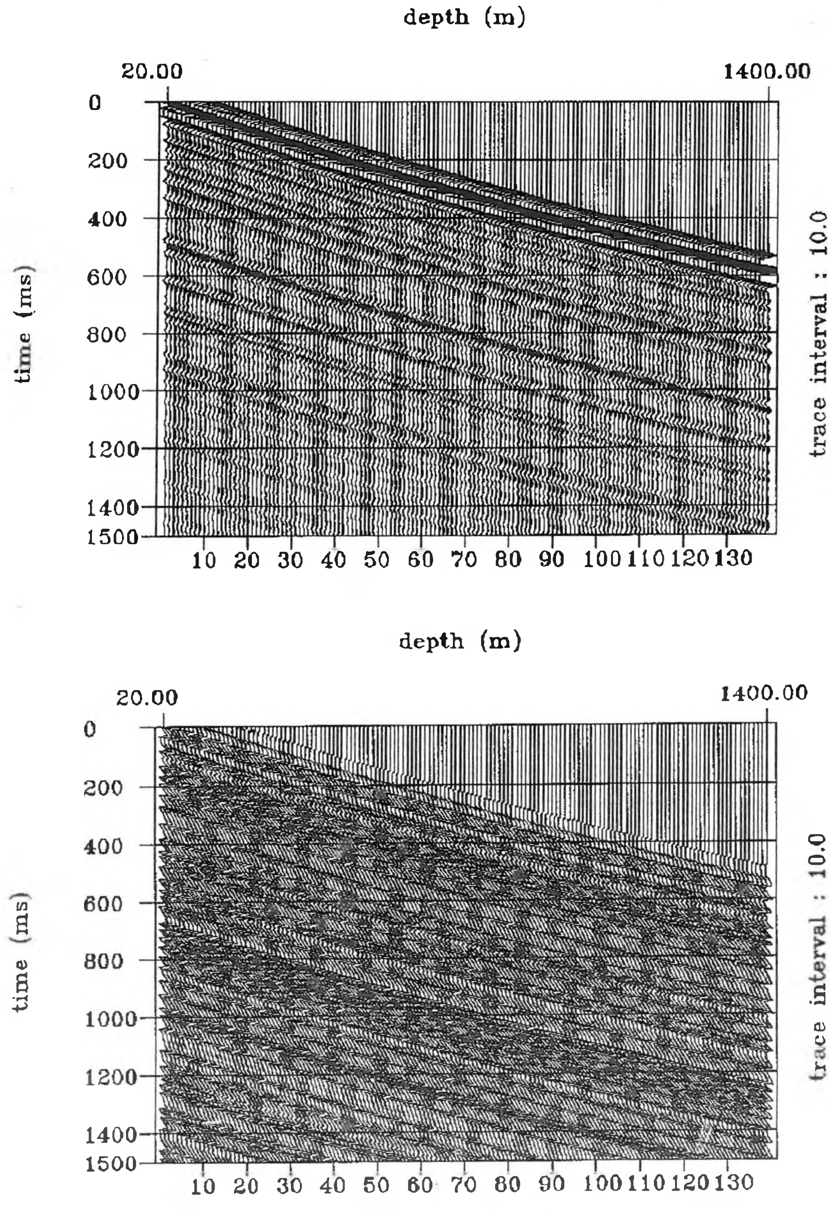


Fig. 2. above: Synthetic zero-offset downgoing wavefield. (wavelet: Klauder with peak frequency of 30 Hz; source depth 10 m). *below*: The corresponding instantaneous phase section

2. ábra. felül: Szintetikus zérus offszetű VSP lefelé haladó hullámtere (wavelet: 30 Hz-es Klauder wavelet; a forrás mélysége 10 m). *alul*: A megfelelő pillanatnyi fázis szelvény estimation uncertainty shown in *Fig. 3b* corresponds to about 66 per cent confidence level, i.e. the true velocity values are between the lower and the upper limits with a probability of about 66 per cent.)

The results show that the velocities estimated from a noise-free synthetic VSP are very close to the true velocities. The mean difference between the two velocity functions is only 1.2 per cent and the estimation uncertainty is less than ±5-6 per cent of the true values. (Throughout this paper *difference* is meant in $\mathcal{L}_1$ norm.)

The most frequent value procedure, which is used here as a tool of statistical processing, maximizes the quotient $n^2_{eff}(\varepsilon)/\varepsilon$, where $n_{eff}(\varepsilon)$ is the so-called number of effective data (the sum of the weights used in the calculation of the most frequent value) and ε denotes the so-called dihesion, expressing the degree of gathering of the data around the most frequent value [STEINER 1988]. *Fig. 3c* illustrates this quotient normalized by n^2, where n is the number of the gradient data used in the course of the statistical processing. This curve informs us about the quality of the VSP data: if $n^2_{eff}(\varepsilon)/\varepsilon$ is very small (e.g. at depths 120 and 330 m), the deviation of the gradient data is large and, as a consequence, the estimation uncertainty increases. On the other hand, when the quality of the data is good, i.e. when the gradient values are close to each other, $n^2_{eff}(\varepsilon)/\varepsilon$ is fairly large and the estimation uncertainty becomes small.

The effect of additive random noise has been investigated by several synthetic VSPs. The standard deviation of the noise was calculated for each trace from the energy of the noise-free VSP trace, random numbers were added to each sample of the synthetic impulse response and the sum was convolved by a Klauder wavelet with peak frequency of 30 Hz. In most cases the inversion procedure used 200 ms wide time gates around the first breaks (i.e. 100 data).

When the additive noise is less than about 6-7 per cent, the differences between the estimated and the true velocities are negligible. In the case of 10 per cent noise the deviation of the estimated velocities from the true velocities becomes visible with a mean difference of 2.6 per cent and the estimation uncertainty is between ±10 and ±20 per cent in most depth intervals (*Fig. 4*). Comparison of the $n^2_{eff}(\varepsilon)/\varepsilon$ plots in Figs. 3 and 4 shows that the quality of the noise-corrupted data is lower than that of the noise-free data, thereby resulting in the increased estimation error and uncertainty. The estimates can be improved if shorter time gates are considered around the first breaks, where the signal to noise ratio remains sufficiently high. If this time interval

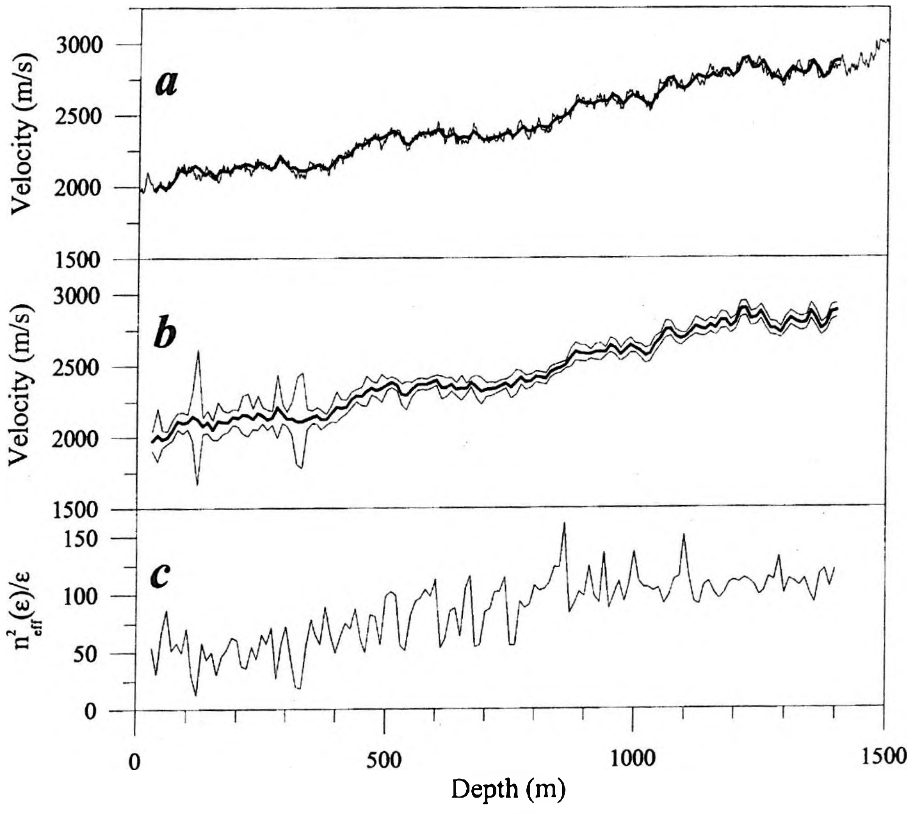


Fig. 3. *(a)* True velocities (thin line) and estimated velocities (thick line) in case of no noise. *(b)* Upper and lower limits (thin lines) of estimated velocities (thick line) illustrating estimation uncertainties. The true velocities are between the two limits with a probability of about 66 per cent. *(c)* $n^2_{eff}(\varepsilon)/\varepsilon$ plot illustrating the quality of the processed VSP data

3. ábra. (*a*) A valódi (vékony vonal) és a becsült (vastag vonal) sebességek zajmentes esetben. (*b*) A becsült sebességek (vastag vonal) bizonytalanságát illusztráló alsó és felső hibahatárok (vékony vonalak). A valódi sebességek kb. 66%-os valószínűséggel esnek a két hibahatár közé. (*c*) Az $n^2_{eff}(\varepsilon)/\varepsilon$ görbe, amely a feldolgozott VSP adatok minőségét jellemzi

is only 50 ms (25 data), the inversion of the VSP corrupted by 10 per cent additive noise again gives fairly good final results: the estimation uncertainty considerably decreases and the mean difference between the estimated and the true velocity functions diminishes to 1.3 per cent.

The gradient of the traveltime curve of the first arrivals is equal to the wave velocity only if the medium is horizontally stratified, even for zero-off-

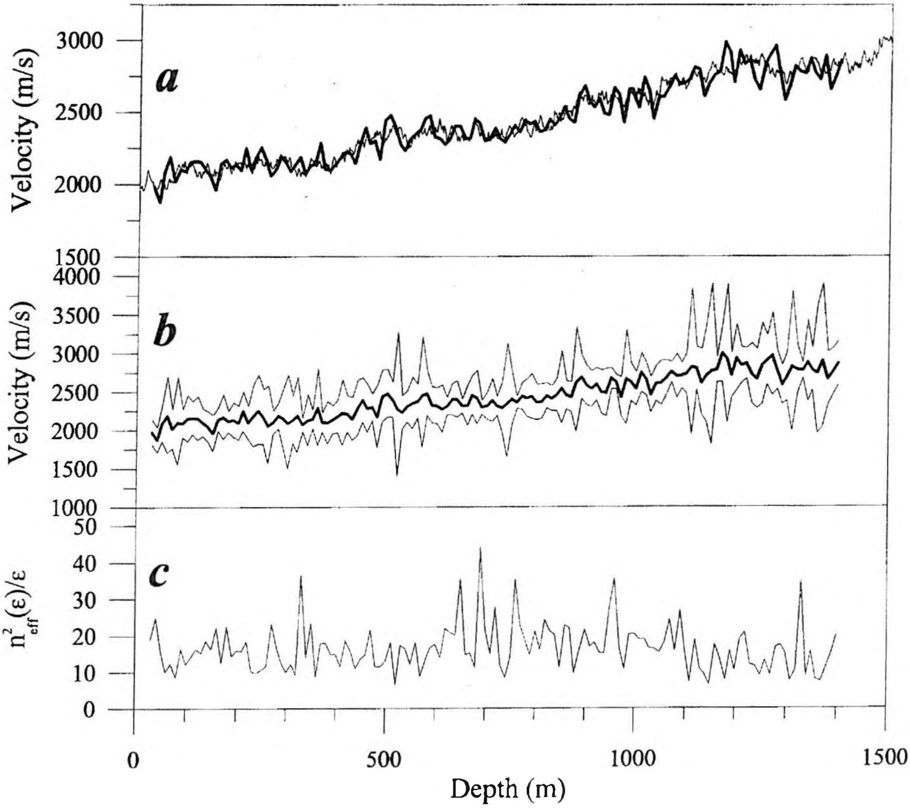


Fig. 4. *(a)* True velocities (thin line) and estimated velocities (thick line) in case of 10 per cent coloured additive noise. *(b)* Upper and lower limits (thin lines) of estimated velocities (thick line) illustrating estimation uncertainties. The true velocities are between the two limits with a probability of about 66 per cent. *(c)* $n^2_{eff}(\varepsilon)/\varepsilon$ plot illustrating the quality of the processed VSP data

4. ábra. (*a*) A valódi (vékony vonal) és a becsült (vastag vonal) sebességek 10%-os additív színes zaj esetén. (*b*) A becsült sebességek (vastag vonal) bizonytalanságát illusztráló alsó és felső hibahatárok (vékony vonalak). A valódi sebességek kb. 66%-os valószínűséggel esnek a két hibahatár közé. (*c*) Az $n^2_{eff}(\varepsilon)/\varepsilon$ görbe, amely a feldolgozott VSP adatok minőségét jellemzi

set VSPs. Thus it is worth investigating how the estimated velocity values differ from the true velocities if the layer boundaries are not horizontal.

In order to carry out this investigation, several two-dimensional geological models were defined with parallel but not horizontal layers, and then noise-free synthetic VSPs were calculated for these models by ray tracing with the following parameters: offset 10 metres; depth of the uppermost geophone 100 metres; geophone spacing 10 metres; and the wavelet is a damped sine function with a peak frequency of 30 Hz. The density is assumed to be constant throughout the models. For the inversion of the synthetic VSPs, a 200 ms wide time gate was used around the first break at each geophone level.

When the layer dips are smaller than about 10-15 degrees, the estimated velocities are practically equal to the true ones (*Fig. 5a*). In the case of layers with a dip of 20 degrees, the differences between the estimated and the true velocity values are somewhat greater, but their mean value is still only 1.3 per cent (*Fig. 5b*). This estimation error systematically increases with depth; this can be explained only by the dip of the layers, which dip was not taken into account during the inversion procedure. In the final analysis, the inversion method can give reliable results even if the dip of the layers is - from a practical point of view - large.

When inverting OVSPs, in the course of the calculation of the correction factor C, a horizontally layered earth model is assumed. In order to investigate how the accuracy of the proposed inversion technique depends on model

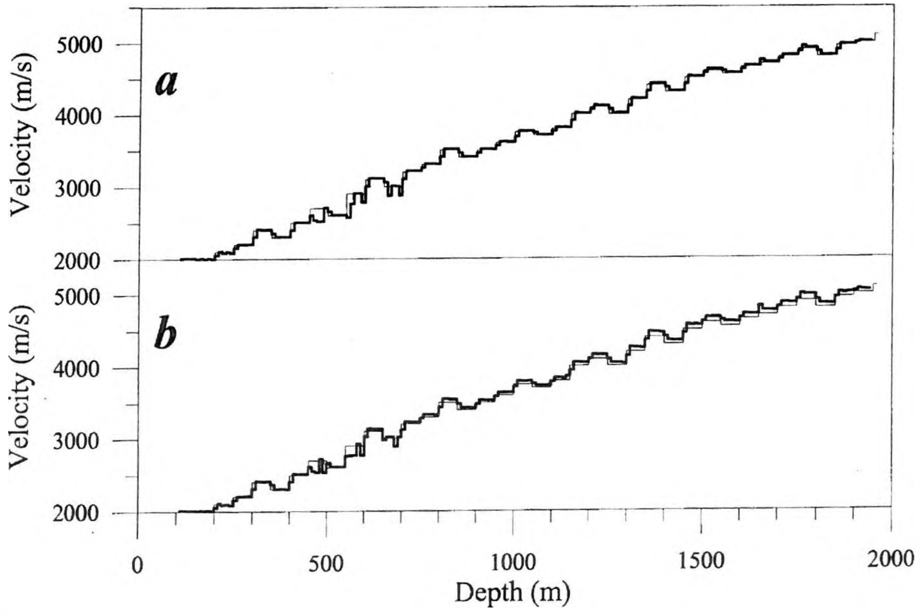


Fig. 5. True velocities (thin line) and estimated velocities (thick line) in case of *(a)* 10-degree and *(b)* 20-degree dipping layers

5. ábra. A valódi (vékony vonal) és a becsült (vastag vonal) sebességek (*a*) 10 fokos és (*b*) 20 fokos dőlt rétegek esetén errors, several noise-free synthetic OVSPs were calculated and then inverted with different source offsets and different layer dips.

The inversion results show that in the case of horizontal layering the estimated velocities are very good approximations to the true ones, independently of the source offset: e.g. for a source offset of 300 m the mean difference between the estimated and the true velocity values is 0.5 per cent only (*Fig. 6a*).

Inversion results for 10-degree dipping layers and an updip source with offset of 300 m are shown in *Fig. 6b*. For a downdip source with offset of 300 m, the results are shown in *Fig. 6c*. The systematic errors between the estimated and the true velocities are due to the nonzero dips of the layers (model errors).

It is also clear from the inversion results that, for a given dip, the estimation error increases with the source offset and if the source is updip, the estimated velocities are greater than the true ones, and if the source is downdip, the estimated velocities are below the true velocities. However, even the greatest systematic errors do not exceed a few per cent.

It should be noted that if the true velocity function consists of relatively thick layers, the estimated velocity function follows the true velocity changes (velocity 'jumps') by insertion of one or two additional steps, even if the estimation is very good. This is due to the fact that the 3×3 point Sobel edge detection operator, during the calculation of the necessary differences, determines the weighted average of the adjacent data [Rosenfeld and Kak 1982]: whereas the traveltime curve of the first arrivals is broken, i. e. where the velocity changes rapidly, the edge detection operator rounds off the traveltime curve. Because of this rounding off, the gradient data do not change rapidly, so the estimated velocity function cannot change rapidly, either.

5. Application to field data

In this section the proposed inversion method is illustrated with a zero-offset VSP and a finite offset VSP measured in different boreholes in Hungary. The source offset for the OVSP is 735 metres. The separated downgoing and upgoing wavefields, the acoustic logs, and the velocity function estimated from the first break times of the zero-offset VSP have been made available for test purposes.

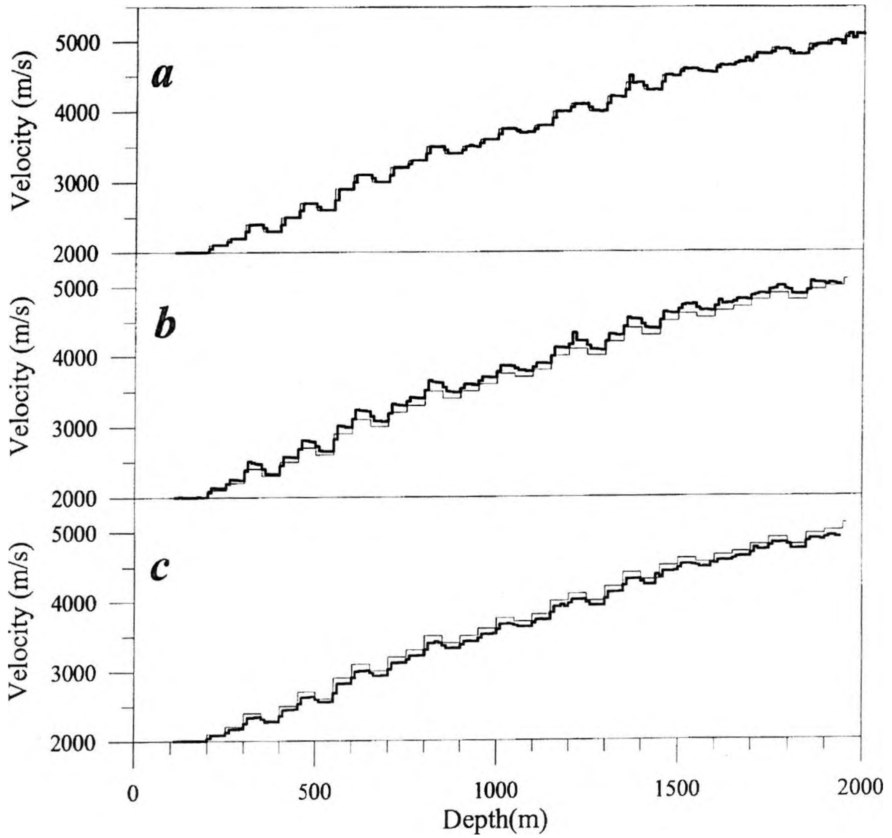
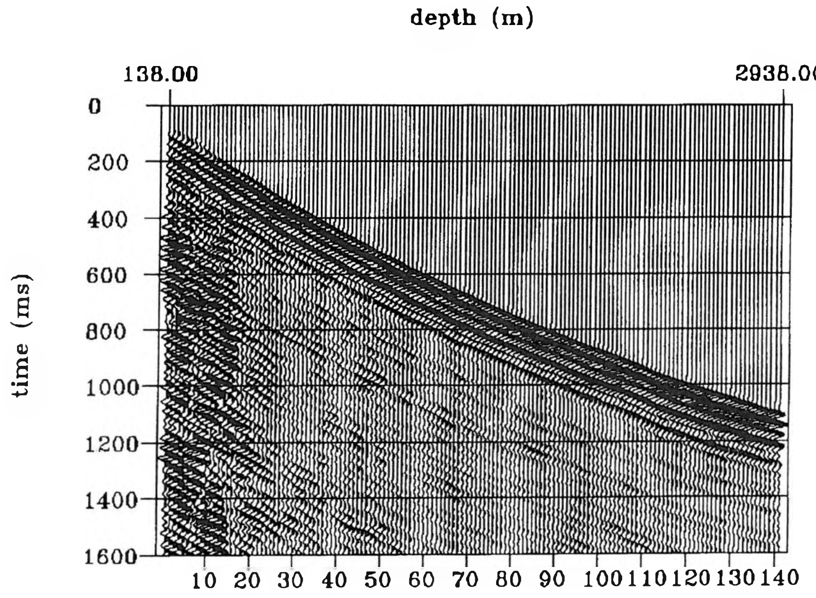


Fig. 6. True velocities (thin line) and estimated velocities (thick line) in case of 300 m source offset: *(a)* horizontal layers, *(b)* 10-degree dipping layers with source updip and *(c)* 10-degree dipping layers with source downdip

6. ábra. A valódi (vékony vonal) és a becsült (vastag vonal) sebességek 300 m-es offszet esetén: (*a*) vízszintes réteghatárok, (*b*) 10 fokos dőlt rétegek, forrás emelkedés irányban, (*c*) 10 fokos dőlt rétegek, forrás lejtés irányban

The downgoing wavefields separated from the two measured VSPs are illustrated in *Fig. 7.* Time gates applied to the OVSP and zero-offset VSP inversions were 100 ms and 150 ms wide, respectively, around the first breaks.

The results of the zero-offset inversion are illustrated in *Fig. 8.* The velocity function calculated from the acoustic log (CVL) and that estimated from the first break times are shown in *Fig. 8a.* The estimated velocity values deviate considerably from the CVL at many depths due to errors in estimates



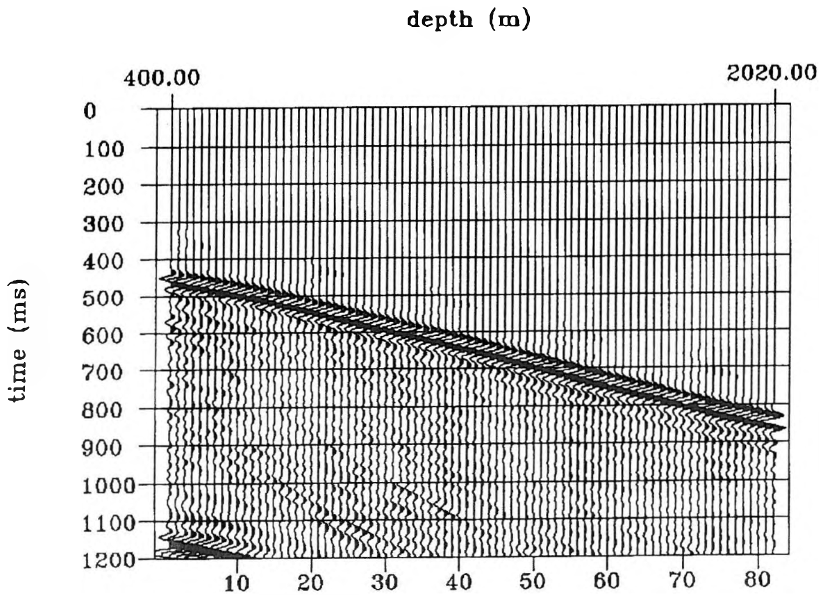


Fig. 7. above: Downgoing wavefield separated from a measured VSP. *below*: Downgoing wavefield separated from a measured OVSP. Source offset 735 m

7. ábra. felül: Egy mért zérus offszetű VSP szelvény lefelé haladó hullámtere. *alul*: Egy mért OVSP szelvény lefelé haladó hullámtere. Offszet: 735 m of the first break times. The mean difference between the two velocity functions is 11.7 per cent, the largest difference is 104.1 per cent.

In *Fig. 8b* the velocities estimated by the proposed method are compared to the CVL: the mean difference between the two sets of velocities is 6.7 per cent, the largest difference is 33 per cent. In most depth intervals the estimation uncertainty does not exceed the value of ± 10 per cent of the estimated velocities. These results prove that the proposed inversion method gives more accurate results than the velocity estimation based on the first break times.

On the basis of the figure it can be observed that the low frequency part of the velocities obtained by the proposed inversion and of that estimated from the first break times are very similar. However, in the depth intervals of 2300-2400 m and 2700-2800 m the results of the inversion deviate considerably from the measured velocity values. These deviations cannot be explained by the uncertainties of the estimates (*Fig. 8c*). In other words, the two velocity functions estimated by two different methods from the same VSP are similar, but - at least in some depth intervals - they are not in accordance with the acoustic log.

Fig. 9 compares the CVL and the velocities calculated from the OVSP data. The mean difference between the two velocity functions is 6.5 per cent, the largest difference is 20.3 per cent. Comparison of *Figs. 8d* and *9c* shows that the quality of the OVSP is lower than that of the zero-offset VSP, resulting in a larger estimation uncertainty (*Fig. 9b*).

The fairly good results illustrated above are partly due to the fact that the geological layers around the boreholes are nearly horizontal, i.e. model errors are small.

6. Conclusions

The inversion technique presented in this paper uses a new method to estimate velocities from zero-offset and finite offset VSPs. After wavelet filtering and wavefield separation, the instantaneous phase section is calculated from the downgoing wavefield. The gradient of the traveltime curve of the first arrivals as a function of depth is estimated from the instantaneous phase section by means of edge detection. In order to obtain the interval velocity from OVSP data, the gradient is multiplied by a correction factor determined by ray trace modelling. The procedure does not need any a priori

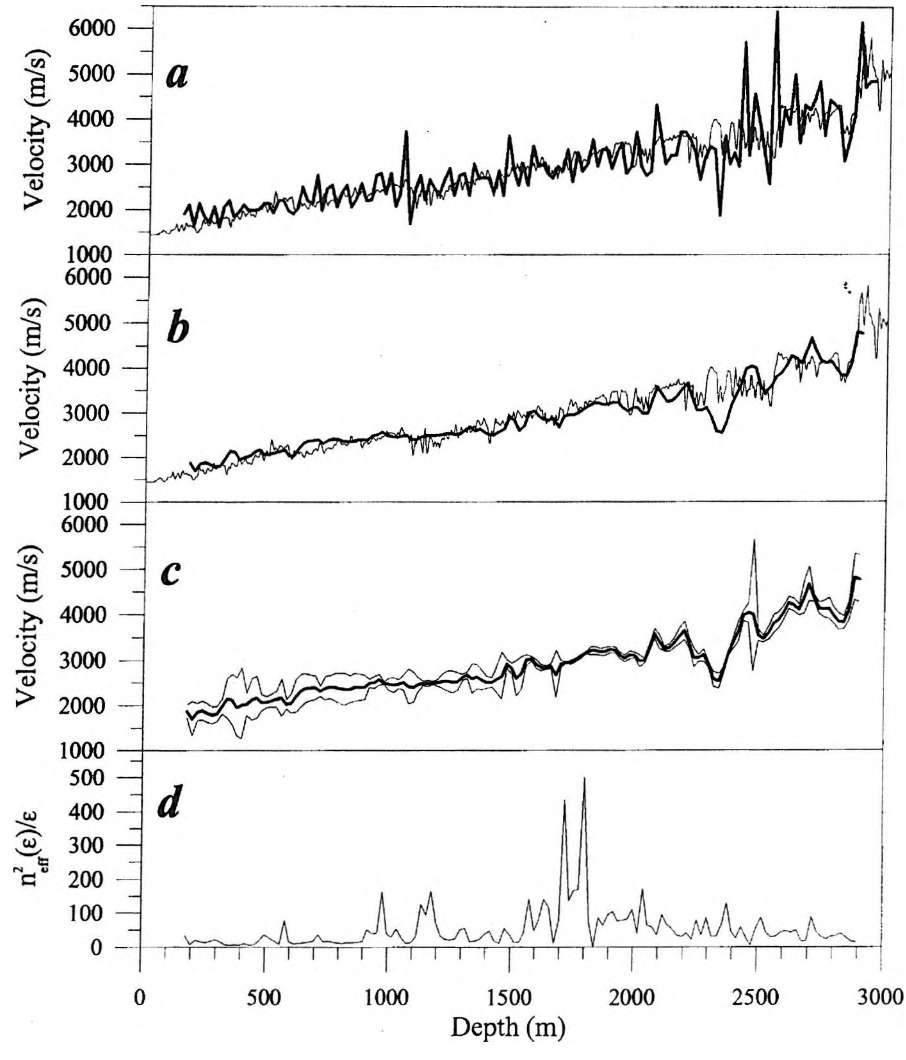


Fig. 8. (a) Velocity log (thin line) and velocities estimated from first break times of the zero-offset VSP (thick line). *(b)* Velocity log (thin line) and velocities estimated by the proposed method (thick line). *(c)* Upper and lower limits (thin lines) of estimated velocities (thick line) illustrating estimation uncertainties. The true velocities are between the two limits with a probability of about 66 per cent. *(d)* $n^2_{eff}(\varepsilon)/\varepsilon$ plot illustrating the quality of the processed VSP data

8. ábra.(a) A sebesség log (vékony vonal) és a zérus offszetű VSP első beérkezési időiből becsült sebességek (vastag vonal). (*b*) A sebesség log (vékony vonal) és a dolgozatban ismertetett eljárással becsült sebességek (vastag vonal). (*c*) A becsült sebességek (vastag vonal) bizonytalanságát illusztráló alsó és felső hibahatárok (vékony vonalak). A valódi sebességek kb. 66%-os valószínűséggel esnek a két hibahatár közé. (*d*) Az $n^2_{eff}(\varepsilon)/\varepsilon$ görbe, amely a feldolgozott VSP adatok minőségét jellemzi

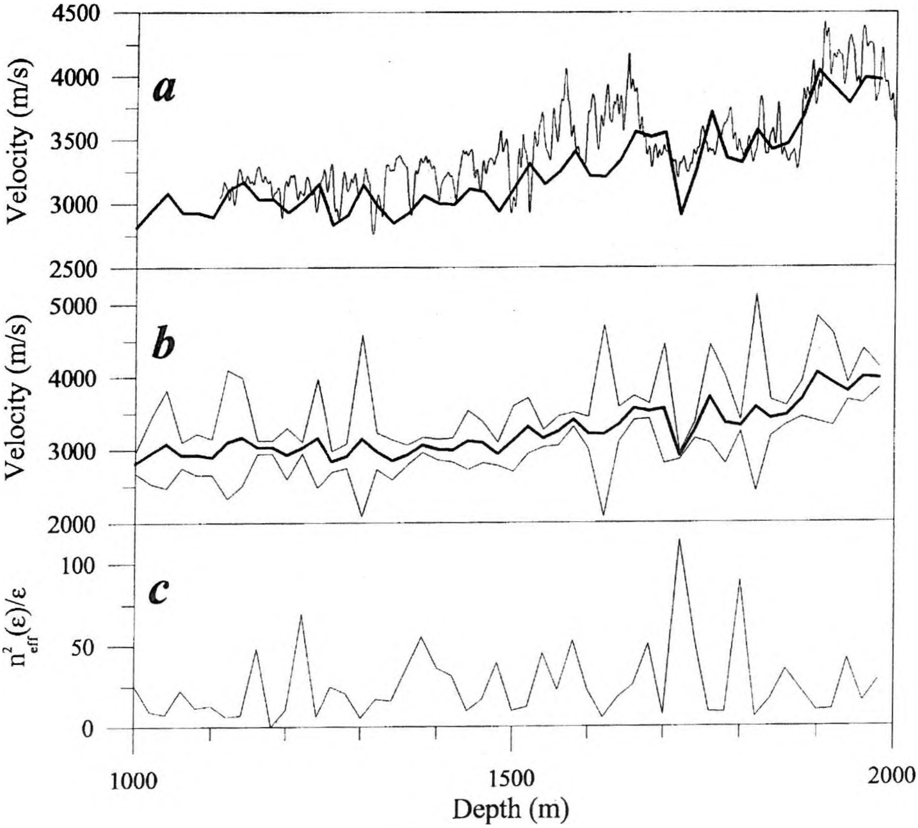


Fig. 9.(a) Velocity log (thin line) and velocities estimated by the proposed method from the finite-offset VSP (thick line). *(b)* Upper and lower limits (thin lines) of estimated velocities (thick line) illustrating estimation uncertainties. The true velocities are between the two limits with a probability of about 66 per cent. *(c)* $n^2_{eff}(\varepsilon)/\varepsilon$ plot illustrating the quality of the processed VSP data

9. ábra.(a) A sebesség log (vékony vonal) és a dolgozatban ismertetett eljárással becsült sebességek (vastag vonal). (*b*) A becsült sebességek (vastag vonal) bizonytalanságát illusztráló alsó és felső hibahatárok (vékony vonalak). A valódi sebességek kb. 66%-os valószínűséggel esnek a két hibahatár közé. (*c*) Az $n^2_{eff}(\varepsilon)/\varepsilon$ görbe, amely a feldolgozott VSP adatok minőségét jellemzi

information. True amplitude processing and initial velocity model are also not required. In addition to the estimated velocity values the estimation uncertainty can also be given. In our implementation the robust and resistant most frequent value procedure was used as a statistical tool. Because of the simple algorithm the inversion procedure is very fast: it can be carried out economically even on personal computers.

Tests of this new technique on synthetic VSPs indicate that the method is very efficient and relatively insensitive to random noise and model errors. Inversions of real zero-offset VSP data prove that the resulting velocities are more accurate than those estimated from the first break times. In the case of OVSPs there is no other method that estimates velocities so accurately and rapidly. However, in complicated geological structures (faults, large layer dips, etc.) model errors can be significant and more testing should be done with field data in areas with good well control.

Appendix

On tracing direct rays with specified end points in layers of constant velocity and horizontal interfaces

When estimating the interval velocities calculation of the raypaths of the first arrivals should be carried out several times. Here, the tracing of direct rays with specified end points is discussed, assuming a horizontally layered earth model, where the horizontal layer boundaries coincide with consecutive geophone positions.

The traveltime t of a direct wave is given by

$$t = \sum_{i=0}^{N} \frac{1}{v_i} \sqrt{\left(x_{i+1} - x_i\right)^2 + d_i^2} ,$$

where x_i is the x-coordinate of the intersection point of the i-th interface and the raypath d_i denotes the thickness of the i-th layer and v_i is the wave velocity in the i-th layer. Since the summation starts with $i=0$, the raypath is defined - disregarding the end points - by N breakpoints. If the borehole is at the zero horizontal coordinate, then x_0 is equal to the offset and x_{N+1} is zero.

Fermat's principle states that traveltime t is stationary for small variations of the raypath. Accordingly, for the actual raypath the following system of nonlinear equations holds:

$$\frac{\partial t}{\partial x_i} = 0 , \quad i.e.$$

$$\frac{1}{v_{i-1}} \frac{x_i - x_{i-1}}{\sqrt{(x_i - x_{i-1})^2 + d_{i-1}^{\ 2}}} + \frac{1}{v_i} \frac{x_{i+1} - x_i}{\sqrt{(x_{i+1} - x_i)^2 + d_i^2}} = 0 \tag{A-1}$$

$$(i = 1,2,\ldots,N)$$

These nonlinear equations can be solved by the Newton-Raphson iteration procedure [e.g. YAKOWITZ and SZIDAROVSZKY 1986.] If $f_i(x_1, x_2, \ldots, x_N)$ denotes the left hand side of Eq. (A-1), then the Jacobian matrix of the above system of non-linear equations is

$$J_{ij}(x_1, x_2, \ldots x_N) = \frac{\partial f_i(x_1, x_2, \ldots x_N)}{\partial x_j} \tag{A-2}$$

Taking into consideration that according to Eq. (A-1) $f_i(x_1, x_2, \ldots, x_N)$ actually depends on x_{i-1}, x_i and x_{i+1} only, the Jacobian matrix of (A-1) is tridiagonal. This involves that the interval velocity estimation method described in this paper is a very fast and efficient procedure and can be carried out economically on personal computers.

REFERENCES

BALCH A. H., LEE M. W. 1984: Vertical Seismic Profiling: Techniques, Applications and Case Histories, IHRDC, 488 p.

GALPERIN E. I. 1974: Vertical seismic profiling: SEG Special Publication No. 12, Tulsa

GANLEY D. C. 1981: A method for calculating synthetic seismograms which include the effects of absorption and dispersion. Geophysics **46**, pp. 1100-1107

GRANT F. S., WEST G. F. 1965: Interpretation Theory in Applied Geophysics. McGraw Hill, New York, 583 p.

GRIVELET P. A. 1985: Inversion of vertical seismic profiles by iterative modeling. Geophysics **50**, pp. 924-930

HARDAGE B. A. 1985: Vertical Seismic Profiling. Part A: Principles. Handbook of Geophysical Exploration, Section I, Vol. 14A, Geophysical Press, London - Amsterdam

LASH C.C. 1980: Shear waves, multiple reflections and converted waves found by deep vertical wave test (vertical seismic profile). Geophysics **45**, pp. 1373-1411

LINES L.R., BOURGEOIS A., COVEY J. D. 1984: Traveltime inversion of offset vertical seismic profiles. A feasibility study. Geophysics **49**, pp. 250-264

MACE D., LAILLY P. 1986: Solution of the VSP one-dimensional inverse problem. Geophysical Prospecting **34**, pp. 1002-1021

PUJOL J., BURRIDGE R., SMITHSON S. B. 1985: Velocity determination from offset vertical seismic profiling data. J. Geophys. Res. **90 (B2)**, pp. 1871-1880

ROSENFELD A., KAK A. C. 1982: Digital Picture Processing. Academic Press, New York, 457 p.

STEINER F. 1988: Most frequent value procedures. Geophysical Transactions **34**, pp. 139-260

STEWART R. R. 1984: VSP interval velocities from traveltime inversion. Geophysical Prospecting **32**, pp. 608-628

TANER M. T., KOEHLER F., SHERIFF R. E. 1979: Complex seismic trace analysis. Geophysics **44**, pp. 1041-1063

URSIN B. 1986: Complete inversion of zero-offset seismic data. Geophysical Prospecting **34**, pp. 1213-1218

YAKOWITZ S., SZIDAROVSZKY F. 1986: An Introduction to Numerical Computations, Macmillan Publishing Company, New York

Képfeldolgozási algoritmusok alkalmazása VSP szelvények inverziójában

WÉBER Zoltán

Jelen dolgozatban az intervallumsebesség vertikális szeizmikus szelvények (VSP) alapján történo meghatározásának egy újszerű megközelítését ismertetjük. A VSP szelvényt kétváltozós képnek tekintve és rajta a képfeldolgozásból jól ismert éldetektáló alforitmust alkalmazva nagy mennyiségű irányadathoz juthatunk. Az azonos mélységhez tartozó irányadatok statisztikus feldolgozásával mind a sebességértékek, mind azok hibái megbecsülhetok.

A javasolt eljárás alkalmazása elott csupán a rutin feldolgozásnak számító jelalak szűrés és hullámtér szétválasztás műveleteit kell végrehajtanunk. Kezdeti sebességmodell definiálására és a valódi amplitúdók visszaállítására nincs szükség. A módszer segítségével offszetes VSP (OVSP) szelvények inverziójára is lehetoség nyílik. Ennek érdekében az OVSP adatok feldolgozása során nyert irányadatokat egy korrekciós faktorral kell megszoroznunk, melynek értékét sugárkövetéses modellezéssel határozzuk meg.

A szintetikus és mért adatokon végzett vizsgálatok azt mutatják, hogy a javasolt módszer nagyon hatékony és viszonylag érzéketlen a véletlen zajokra és a modellhibákra. Az eljárás személyi számítógépen is gazdaságosan megvalósítható.