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Group traveltime estimation by wavelet transform with linear chirp as the basic wavelet

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A method for group traveltime estimation is analysed in this paper. The method is based on the wavelet transform of a seismic signal when the basic wavelet of the wavelet transform is a linear chirp modulated Gaussian. The technique is applied to a synthetic dispersive seismic trace which is calculated for a simple wave guiding sequence. Several tests were carried out on synthetic Love channel waves to compare the result of the above method with the performance of the wavelet transform with a Morlet wavelet as the basic wavelet. Where the frequency-group traveltime curve of the seismic signal can be linearly approximated the wavelet transform with a linear chirp basic wavelet gives a superior result compared to the wavelet transform with a Morlet wavelet as the basic wavelet. It is shown that peaks of the moduli of the wavelet coefficients are sharper, noisy arrivals can be selected easily, and inseparable dispersive arrivals are well resolved.

Keywords: wavelet transform, channel wave, traveltime, dispersion pattern

1. Introduction

There are several methods which are used to gain geological information by the use of dispersive waves. Examples of these are the in-seam reflection technique where guided waves map faults or discontinuities with the help of recompressive filtering [BAKI et al. 1988]; the static correction computation, which is based on velocity dispersion analysis of surface waves [MARI 1984]; or the study of near surface inhomogeneities by group traveltime tomography

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of Rayleigh and Love waves [DOMBROWSKI, DRESEN 1995]. All of these methods and several others require a non-stationary signal analysing tool.

The wavelet transform (WT) has been proved to be a useful way of getting the frequency–group traveltime (or group velocity) function of dispersive waves [PYRAK-NOLTE, NOLTE 1995]. A good analysing method should make the identification of group arrivals easier even in noisy conditions and separate different waves at close arrival times. To achieve these a linear chirp modulated Gaussian is chosen as the basic wavelet for the WT [GROSSMANNet al. 1989].

2. Method

The wavelet transform of a seismic signal $s(t)$ can be given as

$$W(a,b) = \frac{1}{\sqrt{a}} \int g^*\left((t-b)/a\right) s(t) \mathrm{d}t$$

where g^* is the complex conjugate of the scaled (denoted by a) and delayed (denoted by b) version of the basic wavelet. The basic wavelet should meet the admissibility conditions: $g(t)$ is absolutely integrable, square integrable, band limited and has zero mean [SHENSA 1992]. A very simple and commonly used version of $g(t)$ is the Morlet wavelet [MORLET et al. 1982] viz.

$$g(t) = e^{-\frac{t^2}{2}} e^{i2\pi\nu t}$$

where $i^2 = -1$, ν is the central frequency, and $2\pi\nu \geq 5$. The last of these is needed to meet the admissibility conditions. If the data to be analysed can be approximated locally by linear chirp a better choice could be a linear chirp modulated Gaussian as the basic wavelet for the wavelet transform as

$$g(t) = e^{-\frac{t^2}{2}} e^{i2\pi\left(c+\frac{k}{2}t\right)t}$$

where c is the central frequency, k is the rate of change of the instantaneous frequency $c+kt$. Though a new unknown parameter is introduced (k) into the analysis, it can be used to gain additional information and also a better resolution is expected where the linear chirp approximation is valid. These are achieved by repeating the wavelet transform with a set of k values to form a 3D volume, where at each scale the maximum peaks of the moduli of the wavelet coefficients

are selected at the best fit k rate. The additional parameter, which is provided by the choice of the basic wavelet, is the slope of the frequency- group traveltime function derived from that specific k rate at a given a scale.

3. Examples

To analyse the performance of the above method we created synthetic seismograms for a simple model. Two symmetric half spaces separated by a lower velocity layer form a wave guiding sequence. The rocks are homogeneous and nondissipative. We calculated Love arrivals for the following model parameters:

$\rho_0 = 1500$ kg/m^3,
$\beta_0 = 1000$ m/s,
$\rho = 2500$ kg/m3,
$\beta = 2000$ m/s,
$d = 2$ m

where ρ_0 is the density and β_0 is the shear velocity of the seam; ρ is the density and β is the shear velocity of the surrounding rocks; and d is the thickness of the seam. The source and the receiver were placed in the middle of the seam at an offset of 200 m.

4. Group traveltime estimation by WT

Figure 1 shows a noise free dispersive arrival (left) and its WT (middle) where the moduli of the wavelet coefficients are displayed on the c/a–b (frequency and traveltime) plane at $k = 0$ in the basic wavelet (the result is equivalent to a WT with a Morlet wavelet). The figure shows the k–b slice of the 3D volume at a given a scale ($c/a = 240$ Hz), too (right). This a scale was chosen from the linear section of the frequency–group traveltime curve. $k = 0$ for the first displayed channel and k increases channel by channel. The WT has the sharpest peak at about $k = 800$ Hz/s chirp rate. This occurs when the seismic signal is more closely approximated locally by a linear chirp with the given k rate than with $k = 0$. It makes the group traveltime estimation more accurate and also provides additional information since the k rate gives the slope of the frequency–group traveltime curve at the a scale.

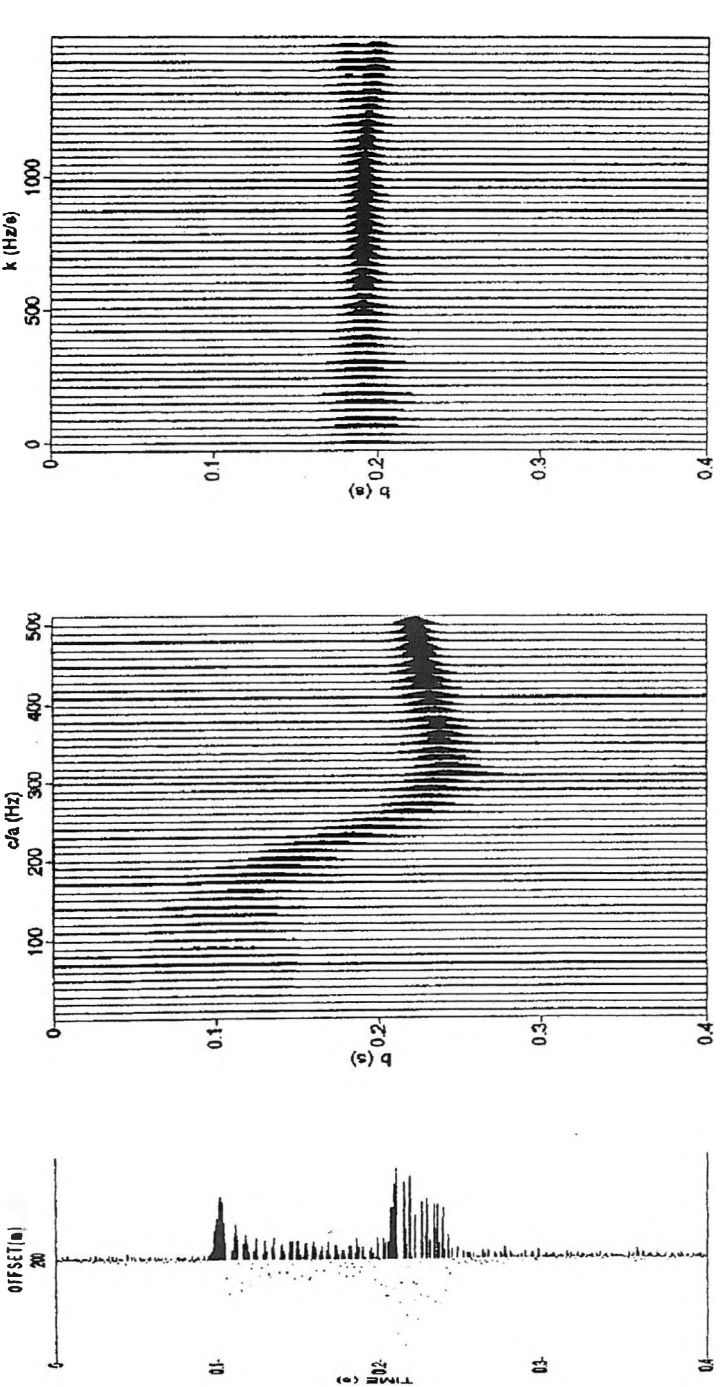

Fig. 1. Noise free synthetic Love channel wave (left), its wavelet transform (middle) and the *k*–*b* slice (right)
1. ábra. Zajmentes szintetikus Love-típusú csatornahullám (balra), wavelet transzformáltja (középen) és a *k*–*b* metszet (jobbra)

Figure 2 shows a noisy dispersive arrival (left) and its WT at $k = 0$ in the basic wavelet (middle), on the *c/a–b* plane (similarly to Figure 1.). The noise is random and its distribution is Gaussian. The level of the noise was chosen to cover the dispersive arrival. Based on the wavelet transform with a Morlet wavelet as the basic wavelet the theoretical frequency–group traveltime curve is hardly recognizable due to the added strong random noise, but the *k–b* slice of the 3D volume at the same *a* scale as previously (right) still enables us to select the right group traveltime at a specific *k* rate accurately enough.

Figure 3 shows two dispersive arrivals when a 200 m offset and a 250 m offset trace were added (left). With this simple example we demonstrate the resolving power of the method. The WT (middle) at $k = 0$ in the basic wavelet, on the *c/a–b* plane (similarly to the previous figures) were able to differentiate between the two separate arrivals only at the higher frequency range. The *k–b* slice of the 3D volume at the same *a* scale as previously (right) where the WT with the Morlet wavelet showed no separate arrivals (middle) the WT with a linear chirp as the basic wavelet at different *k* rates was able to produce distinct arrivals.

5. Conclusions

The group traveltime of a seismic signal can be analysed by the wavelet transform with linear chirp as the basic wavelet. Repeating the WT at different chirp rates a 3D volume can be created where the amplitude peaks at each scale are sharper at the frequency rates which correspond to the linear approximation of the frequency–group traveltime curve at the given frequencies. In this way we get the slope of that curve, the selection is more accurate on noisy data and different arrivals are better separated even at close arrival times than by the WT with a Morlet wavelet as the basic wavelet. Though the examples were calculated on dispersive seismic traces the method can be used whenever the linear approximation of the frequency–group traveltime function of a signal is valid.

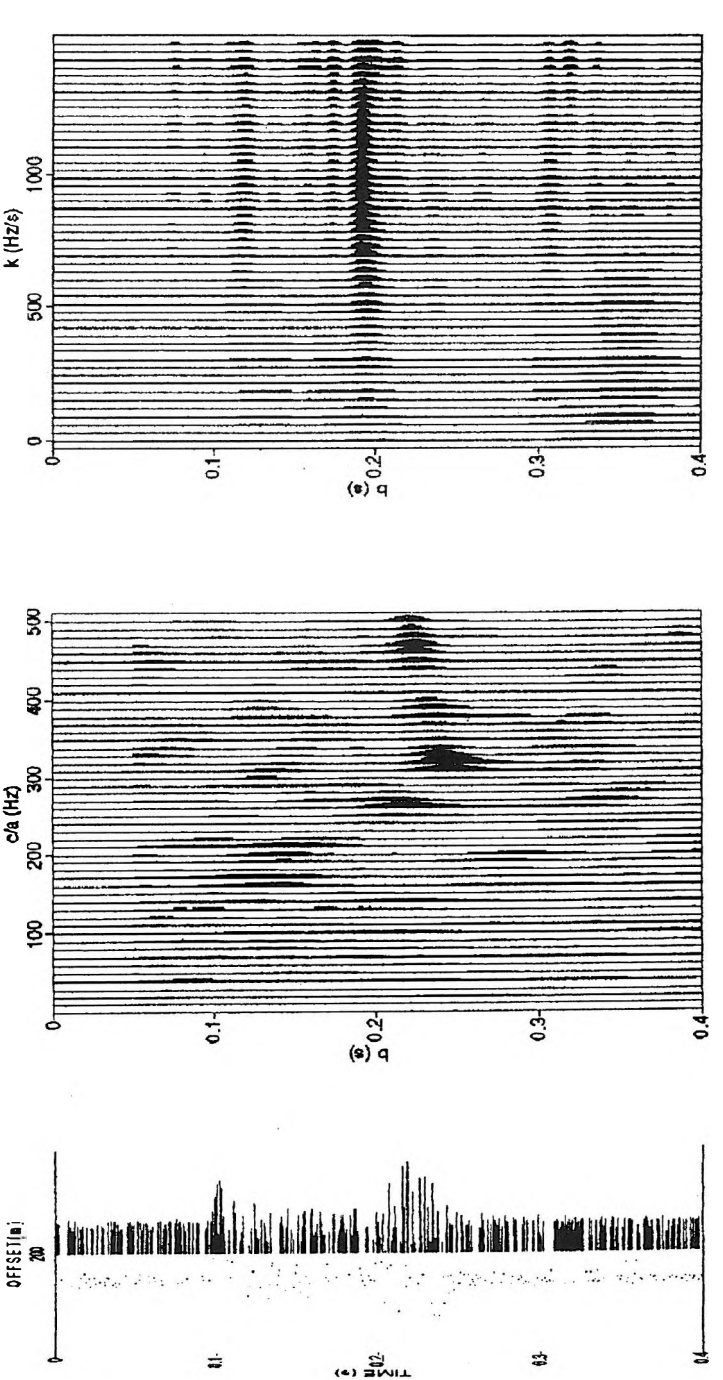

Fig. 2. Noisy synthetic Love channel wave (left), its wavelet transform (middle) and the *k–b* slice (right)

2. *ábra.* Zajos szintetikus Love-típusú csatornahullám (balra), wavelet transzformáltja (középen) és a *k–b* metszet (jobbra)

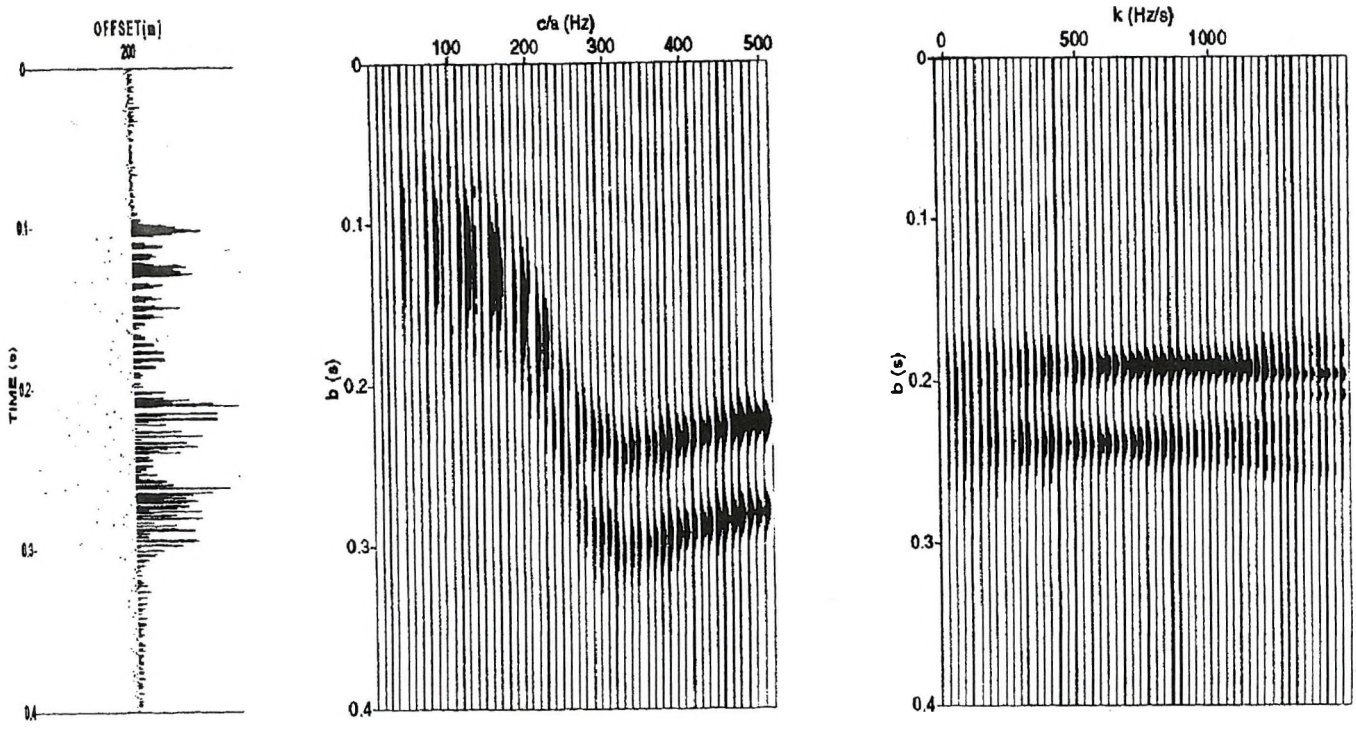

Fig. 3. Added synthetic Love channel waves (left), the wavelet transform (middle) and the *k–b* slice (right)
3. ábra. Szuperponált Love-típusú csatornahullámok (balra), wavelet transzformáltjuk (középen) és a *k–b* metszet (jobbra)

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Csoport futási idő meghatározás wavelet transzformált segítségével lineárisan változó frekvenciájú alap wavelet alkalmazásával

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A dolgozat wavelet transzformáció segítségével történő futási idő meghatározást mutat be. A diszperzív szeizmikus hullám analízisét lineárisan változó frekvenciájú jellel modulált Gauss-típusú alap wavelet alkalmazásával végzi. A módszert egy egyszerű hullámvezető modell alapján számítható szintetikus diszperzív szeizmikus csatornán vizsgálja. Többféle tesztet végez Love-típusú csatornahullámokon a közölt módszer és a már régebben is alkalmazott wavelet transzformációs eljárás összehasonlítására, melynél az alap wavelet a Morlet-típusú wavelet. Bemutatja,

hogy ahol a szeizmikus jel frekvencia- csoport futási idő függvénye egyenessel közelíthető, ott a lineárisan változó frekvenciájú jellel modulált Gauss-típusú alap wavelet használata jobb eredményt nyújt, mint a Morlet-típusú wavelet: a wavelet transzformáció eredményeként kapott wavelet koefficiensek képe élesebb, zajos beérkezések könnyebben azonosíthatók és az időben közeli beérkezések jól felbonthatók.