Magyar Állami Eötvös Loránd Geofizikai Intézet
GEOFIZIKAI KÖZLEMÉNYEK
26

APPROXIMATE GRAPHICAL AND NUMERICAL DETERMINATION OF DENSITIES CORRECTED FOR BOREHOLE DIAMETER

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Borehole diameters in exploratory drillings vary over a wide range. It should always be kept in mind that the gamma-gamma (density) logs recorded in such holes are seriously affected by the diameter; in view of this, the determination of densities is subject to great errors.

This paper presents approximate graphical and numerical solutions for borehole diameter correction and for determining the corrected densities. The mathematics will be sufficiently simple to allow for interpretation by desk-top computers and, also, to serve as a methodological basis for the development of a ϱ-processor yielding borehole compensated values *in situ*.

ELGI's up-to-date Modelling Laboratory (with its comprehensive stock of models of natural rock constituents) enabled us to study experimentally the effect of borehole diameter on different kinds of sondes. The graphical and numerical relationships to be presented in the following have been derived on the basis of such measurements.

Performed on standard reference series of layers and in holes of different diameters, measurements have shown that the master curves of the sondes

$$N=f(\varrho, d)$$

exhibit behaviour which differs depending on borehole diameter.

The connection between the slope m characterizing the master curve and diameter d is shown in Fig. 1. The connection is linear for both short and long probes.

Having determined the slopes m_{sh} and m_l for the short and long probes respectively, we studied the connection between the basis equation (ANDRÁSSY 1978) and borehole diameter. The basis equation can be written as

$$Y_{sh} - Y_l = (m_{sh} - m_l)\varrho + (b_{sh} - b_l)$$

where Y_{sh}, Y_l are logarithms of the counts, normalized to water, measured by the short and long probes respectively;

$M = m_{sh} - m_l$ slope computed from the short and long probes;

$B = b_{sh} - b_l$ represents a constant computed from the ordinate-crossings of the short and long probes.

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As an example, Table I shows the m_{sh}, m_l and $B=b_{sh}-b_l$ values computed from the master curves of the KRGG–2–120–60sY type two-detector sonde:

Table I.

d	m_{sh}	m_l	$M=m_{sh}-m_l$	$B=b_{sh}-b_l$
86	−0.2872	−1.9621	1.6749	−1.7094
150	−0.2837	−1.8515	1.5678	−1.5596
214	−0.2781	−1.7315	1.4338	−1.4219

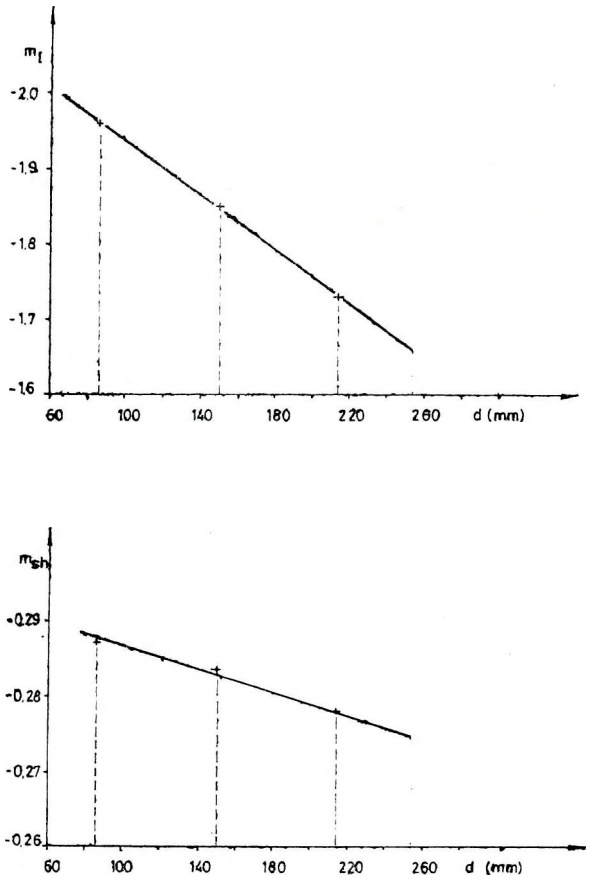

Fig. 1. Connection between borehole diameter (d) and slopes (m) of the master curves for the short and long sondes, respectively

1. ábra. Összefüggés a fúrólyuk-átmérő (d) és a hosszú, ill. a rövid szonda iránytangens értékei között (m_l, ill. m_{sh})

Рис. 1. Зависимость между диаметром скважины и значениями углового коэфициента длинных (m_l) и коротких зондов (m_{sh})

The dependence of the M factor on borehole diameter d is shown by Fig. 2. The broken line denotes the values of B.

The linear relations shown in Figs. 1 and 2 can be used for hole-diameter correction. For the computations we first have to adopt some standardizing conditions. Most sondes are generally applied in holes whose diameters range from $d=86$ mm to $d=214$ mm. We consider $d=150$ mm as the *basis diameter* and derive the corresponding basis equation. Next, we

determine the borehole diameter corrections $\Delta\varrho$, for diameters smaller and greater, respectively, than $d=150$ mm.

Expressing the density from the basis equation for the $d=150$ mm case, we have

$$\varrho=\frac{Y_{sh}-Y_l}{M}+1 \tag{1.1}$$

where, by Table I, $M=1.5678$. The master curve of the sonde for $d=150$ mm is shown in Fig. 3.

The borehole diameter correction will be determined on the basis of the linear connection between slope M and diameter d.

Using the data obtained for borehole diameters greater or less than the basis diameter (test and routine measurements) in the above formula yields apparent density values ϱ_{app}. It is obvious that the deviations between densities are due to the difference of borehole diameters–these are the borehole diameter corrections $\Delta\varrho$ sought. For $d>150$ mm, $\Delta\varrho$ is positive, for $d<150$ mm it is of negative sign.

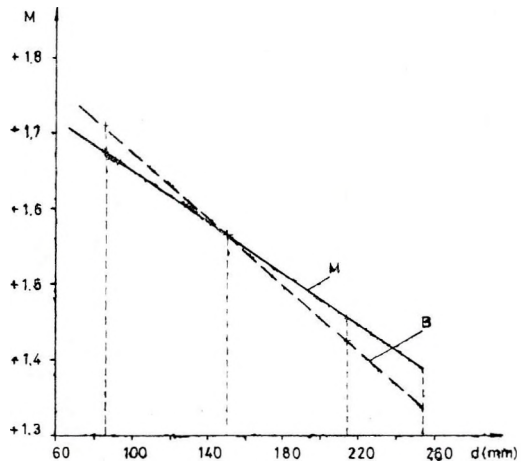

Fig. 2. Connection between borehole diameter and the constants M and B figuring in the basis equation

2. ábra. Összefüggés a fúrólyukátmérő és a bázisegyenletben szereplő M és B értéke között

Рис. 2. Зависимость между диаметром скважины и значениями M и B в базисном уравнении

Figure 4. shows the relationship between density ϱ, borehole diameter d and the correction $\Delta\varrho$. Apparent densities are displayed on the horizontal axis, corrections on the vertical one. The parameter of the pencil of straight lines is the borehole diameter d. The limits corresponding to the density error $\Delta\varrho=\pm0.05$ g/cm^3 are denoted by the dotted lines. Note that for the basis diameter $d=150$ mm the apparent density is equal to its diameter corrected value since the $\Delta\varrho$ correction is evidently zero.

Let us now describe the above diagram mathematically. Generally, we proceed as follows. We determine the equations of any two different lines, say the two denoted by heavy lines in Fig. 4. In the present case the corresponding diameters are $d=86$ mm and $D=214$ mm.

$$\Delta\varrho=-m_d\varrho+b_d \qquad (d=86\text{ mm}) \tag{1.2}$$

$$\Delta\varrho=+m_D\varrho-b_D \qquad (D=214\text{ mm}) \tag{1.3}$$

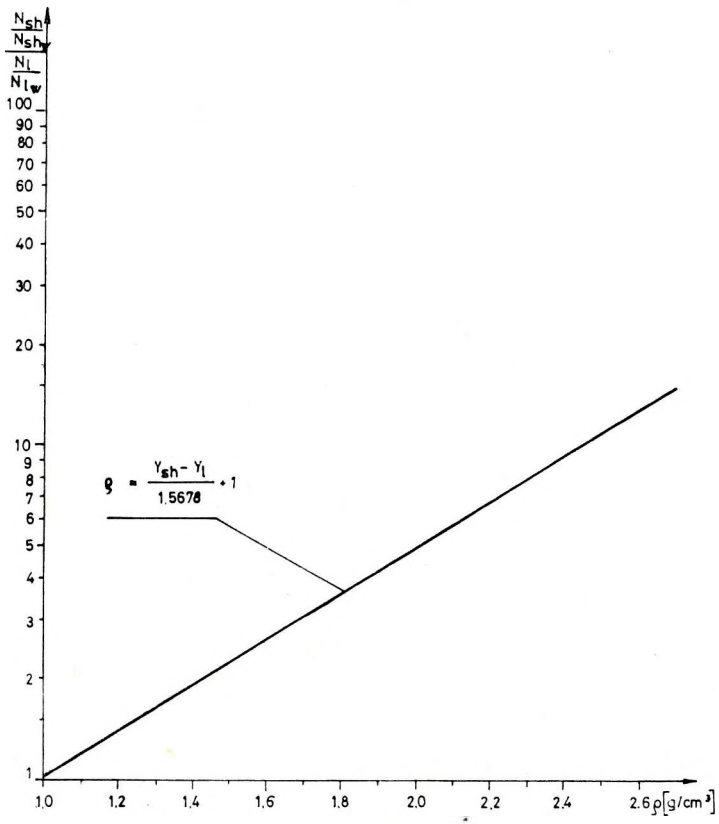

Fig. 3. Master curve of the two-detector KRGG—2—120—60 sY type sonde for borehole diameter d=150 mm

3. ábra. KRGG—2—120—60 sY típusú két-detektoros szonda hitelesítő görbéje d=150 mm-es átmérőre

Рис. 3. Атестационная кривая двухдетекторного зонда типа KRGG—2—120—60 sY при диаметре скважины d=150 мм

The straight lines (1.2) and (1.3) cross the horizontal axis ϱ_{app} at the respective points b_d/m_d and b_D/m_D. Because of the inherent error of the determined parameters, these abscissae could be slightly different. In such a case, we take their mean. For the above-mentioned sonde the abscissa becomes 1.2996 g/cm³.

Knowing the slopes m_d and m_D in Eqs. (1.2) and (1.3), and assuming a linear variation, we compute the increase of the slope for a change of 1 mm in the diameter:

$$\eta = \frac{m_D - (-m_d)}{D - d} \tag{1.4}$$

If we apply the tangent theorem for the triangle of Fig. 4 the borehole diameter correction $\Delta\varrho$ becomes

$$\Delta\varrho=(d-150)\left[\frac{m_d+m_D}{D-d}\right](\varrho_{app}-\varrho_{abs.\ cross}) \tag{1.5}$$

where

$$\varrho_{abs.\ cross}=-\frac{m_D b_d+m_d b_D}{2m_d m_D}.$$

The sign of $\Delta\varrho$ can be either positive or negative, depending on whether d is greater or less than 150 mm.

For the above-discussed KRGG–2–120–60sY type sonde the algorithm of density determination is given by the combined form of Eqs. (1.1) and (1.5). Substituting the empirical parameters, we obtain the formula

$$\varrho_{actual}=\left\{\frac{Y_{sh}-Y_l}{1.5678}+1\right\}\pm(d-150)\{0.001106(\varrho_{app}-1.2996)\} \tag{1.6}$$

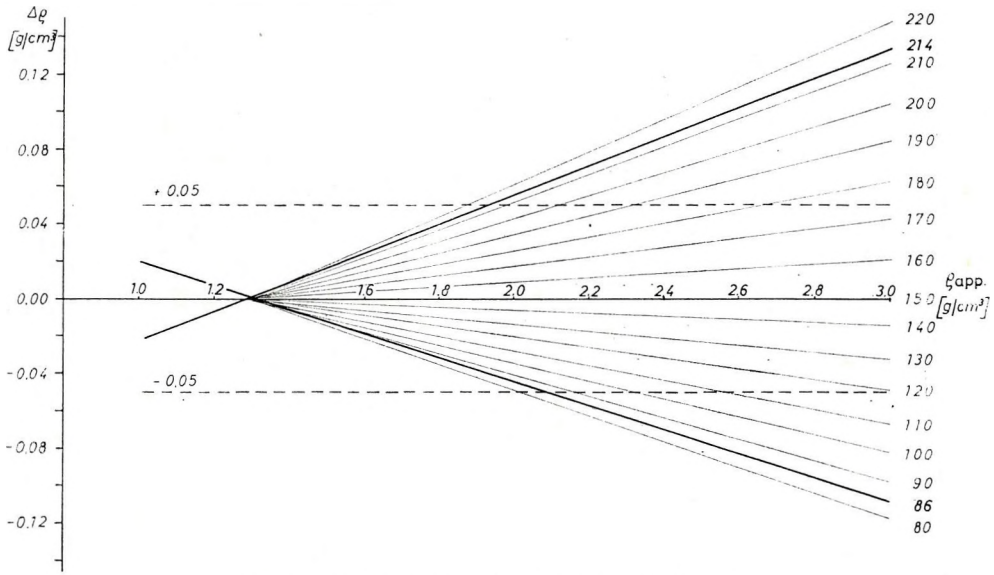

Fig. 4. Diagram for borehole diameter correction $\Delta\varrho$

4. ábra. Diagram a $\Delta\varrho$ átmérő korrekció meghatározására

Рис. 4. Диаграмма для определения коррекции на диаметр $\Delta\varrho$

Practical examples

The practical application of the diagrams (Figs. 3, 4) and the numerical formula (1.6) will be shown by two examples. It will be assumed that the borehole diameter is the single quantity affecting the measured values.

For the Nagyegyháza (Cs–260) drilling we applied the graphical and numerical procedures (by means of the SR–56 desk-top calculator) at 17 points. Gamma-gamma and caliper logs are shown in Fig. 5. In the investigated part of the section, the borehole diameter varies from 110 mm to 185 mm. Results of the graphical and numerical calculations are in fair agreement, see Table II. Diameter correction $\Delta\varrho$ is always negative (except for layers No. 3, and No. 7) since the diameters are always less than 150 mm.

For the Mány—132/B drilling the computation was performed for 23 layers, by a desk-top calculator. The layers to be examined were selected on the basis of lithology, and the different well-logs, to have divers geological and petrophysical characteristics.

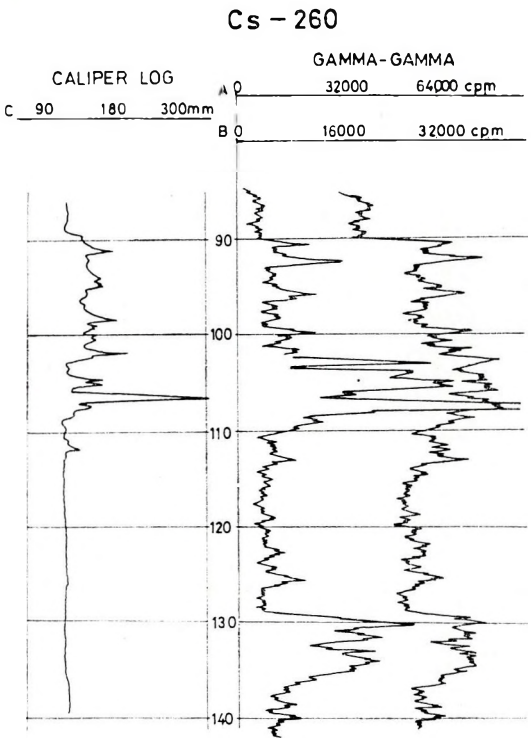

Fig. 5. Well-log material for a section of the Cs—260 hole
A — gamma-gamma log recorded with short sonde
B — gamma-gamma log recorded with long sonde
C — caliper log

5. ábra. Cs—260 fúrás egy szakaszának gamma-gamma és lyukbőség szelvényanyaga
A — rövid szondával felvett gamma-gamma
B — hosszú szondával felvett gamma-gamma
C — lyukbőség szelvény

Рис. 5. Гамма-гамма профили и кавернограммы части скважины Cs-260
A — Гамма-гамма профиль снятый с коротким зондом;
B — Гамма-гамма профиль снятый с длинным зондом;
C — Кавернограмма

From the log an about 100 m long section (from 450 m to 560 m) has been selected. The complete logging material consists of resistivity, gamma-gamma, neutron-neutron, acoustic and caliper logs. Dolomite is overlain by marl, sandy marl, sandy shale, shaly sand, shale and sandy formations of varying thickness. The dolomite-marl boundary is at a depth of 534 m.

Corrected ϱ values and borehole corrections ($\Delta\varrho$) are shown in Table III. From the computed 23 layers, presented in Table III, ten layers designated as 1–10 are shown in Fig. 6.

$\Delta\varrho$ is positive in all cases. The densities ϱ range between 2.69 and 2.74 g/cm^3 for the dolomite; between 2.14 and 2.33 g/cm^3 for the shaly-sand and marl formations. Borehole corrections do not exceed 0.05 g/cm^3.

Table II

Layer No.	Caliper d [mm]	Graphically determined		Computed	
		ϱ [g/cm³]	$\Delta\varrho$ [g/cm³]	ϱ [g/cm³]	$\Delta\varrho$ [g/cm³]
1	138	2.18	−0.004	2.18	−0.0118
2	138	2.29	−0.008	2.29	−0.0133
3	185	1.93	+0.028	1.99	+0.0258
4	145	2.16	−0.004	2.17	−0.0048
5	145	1.76	−0.002	1.77	−0.0026
6	150	2.17	0.000	2.18	0.0000
7	155	1.89	0.004	1.90	0.0033
8	150	2.18	0.000	2.20	0.0000
9	115	2.06	−0.026	2.04	−0.0300
10	115	2.06	−0.026	2.04	−0.0299
11	130	2.04	−0.014	2.03	−0.0166
12	110	2.20	−0.034	2.18	−0.0406
13	110	2.26	−0.038	2.23	−0.0430
14	110	2.09	−0.030	2.07	−0.0357
15	110	1.95	−0.026	1.93	−0.0292
16	110	2.32	−0.080	2.28	−0.0455
17	110	2.31	−0.038	2.29	−0.0459

Conclusions

It is possible to obtain a theoretically deduced method that can be checked in practice for the rapid and accurate determination of density values corrected for borehole diameter. The computation is easily carried out even under field conditions by a small desk-top calculator. Formula (1.6) is not restricted to KRGG–2–120–60sY type sondes; similar formulae can easily be derived for any other two-detector gamma-gamma sondes. If, say, a KRGG–2–120–43sY type sonde is considered, the standardization conditions should be changed and $d=86$ mm should be taken as the basis diameter. Of course, for well-collimated systems with a sufficiently large diameter the borehole effect becomes negligible.

It is hoped that the procedure presented will be of value for the quantitative interpretation of gamma-gamma logs recorded in exploratory drillings.

Table III

Borehole: Mány-132/B
Sonde: KRGG—2—120—60 sY,№7636 $N_{hs_w} = 84\,320$ cpm
Isotope: Cs^{137}14.7 mCi $N_{l_w} = 62\,712$ cpm

№	Layer	Depth [m]	Caliper d [mm]	$a_{sh}=13$ cm		$a_l=38$ cm			ϱ_b [g/cm³]	$\Delta\varrho$ [g/cm³]
				N_{sh} [cpm]	$\frac{N_{sh}}{N_{sh_w}}$	N_l [cpm]	$\pm\sigma_1$ [cpm]	$\frac{N_l}{N_{l_w}}$		
1	dolomite	546.9—548.4	155	52 309	0.6204	2665	163	0.0425	2.72	0.0078
2	dolomite	545.4—546.9	155	54 032	0.6408	2962	172	0.0472	2.67	0.0075
3	dolomite	542.6—544.4	155	51 078	0.6058	2517	159	0.0401	2.74	0.0079
4	dolomite	540.8—542.6	155	52 001	0.6167	2665	163	0.0425	2.71	0.0078
5	dolomite	534.9—537.8	155	52 432	0.6218	2769	166	0.0442	2.69	0.0077
6	shale-marl	525.4—527.4	165	66 463	0.7882	8145	285	0.1299	2.16	0.0141
7	shale-marl	523.0—524.8	155	68 617	0.8138	8589	293	0.1370	2.14	0.0046
8	shale-marl-sand	519.0—521.0	165	62 155	0.7371	6516	255	0.1039	2.26	0.0158
9	shale-marl	501.6—506.0	198	62 463	0.7408	6694	259	0.1067	2.29	0.0497
10	shale-marl	493.0—498.0	180	62 770	0.7444	7108	267	0.1133	2.23	0.0299
11	shaly sand	431.2—433.8	150	60 616	0.7189	6220	249	0.0992	2.26	0.0000
12	shaly sand	420.0—423.6	160	61 847	0.7335	6220	249	0.0992	2.29	0.0108
13	sand, shaly sand	399.0—403.4	158	58 155	0.6897	5775	240	0.0921	2.29	0.0087
14	shaly sand	373.0—380.0	162	60 616	0.7189	5924	243	0.0945	2.31	0.0132
15	shaly sand, shale-marl	340.0—344.0	155	62 463	0.7408	6368	252	0.1015	2.27	0.0054
16	shale	325.0—327.2	158	59 694	0.7079	5657	238	0.0902	2.32	0.0090
17	shale-marl	318.4—320.0	180	57 847	0.6860	5479	234	0.0874	2.33	0.0168
18	sandy shale, marl	314.6—316.6	165	60 924	0.7225	5998	245	0.0956	2.31	0.0164
19	sand, shale-marl	242.6—252.2	155	60 924	0.7225	6220	249	0.0992	2.27	0.0053
20	shale-marl	217.8—220.0	155	63 078	0.7481	6812	261	0.1086	2.25	0.0155
21	sand	204.8—208.2	155	61 540	0.7298	6886	262	0.1098	2.21	0.0050
22	sand	183.6—187.4	158	62 770	0.7444	6960	264	0.1110	2.22	0.0081
23	marl	150.0—155.8	165	55 386	0.6569	4072	202	0.0649	2.50	0.0195

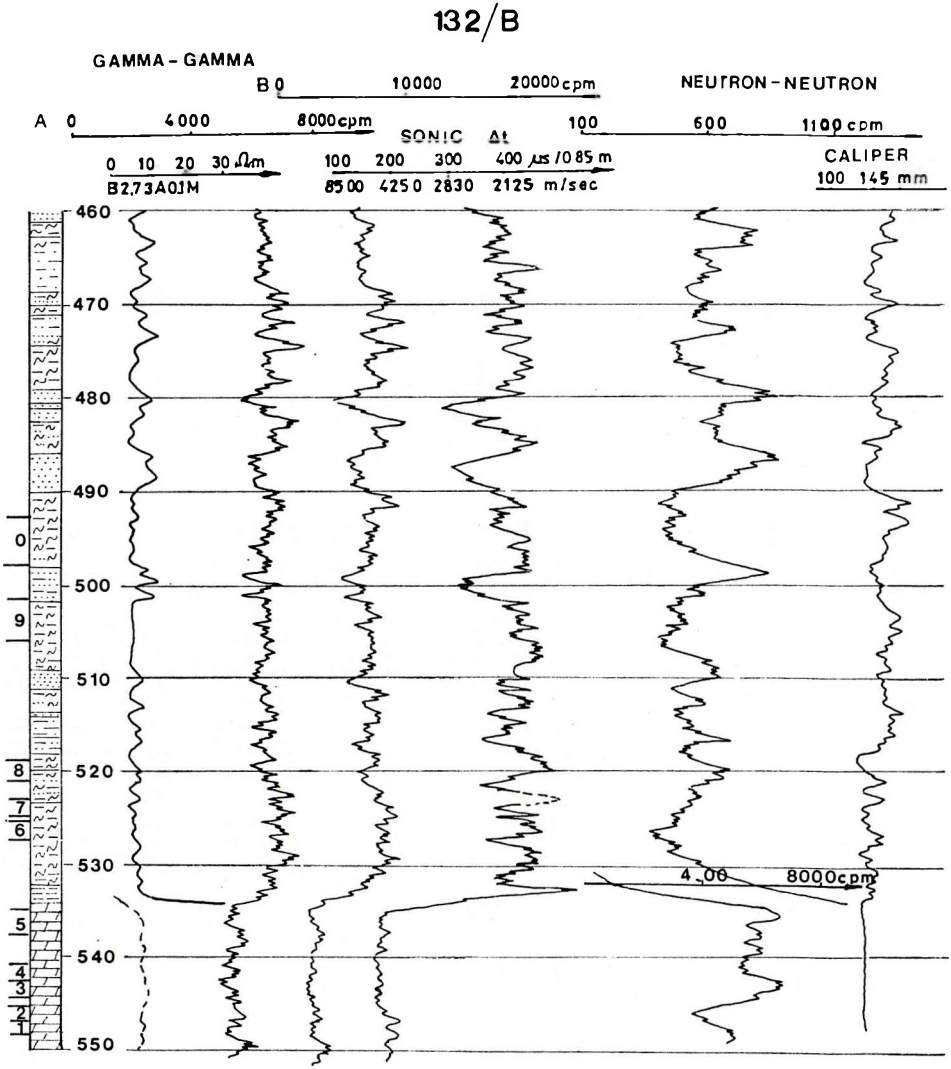

Fig. 6. Lithology and well-log section for part of the Mány—132/B borehole
A — gamma-gamma log recorded with short sonde
B — gamma-gamma log recorded with long sonde

6. ábra. Mány—132/B fúrás egy szakaszának földtani és komplex karotázs szelvényanyaga
A — rövid szondával felvett gamma-gamma
B — hosszú szondával felvett gamma-gamma

Рис. 6. Геологические и комплексные каротажные профили части скважины Мань-132/В
А — Гамма-гамма профили снятие коротким зондом;
В — Гамма-гамма профили снятиедлинным зондом

ANDRÁSSY LÁSZLÓ

KÖZELÍTŐ GRAFIKUS ÉS MATEMATIKAI MEGOLDÁS A TÉRFOGATSÚLYÉRTÉKEK FÚRÓLYUKBAN TÖRTÉNŐ KÖZVETLEN MEGHATÁROZÁSÁRA, A FÚRÓLYUKHATÁS FIGYELEMBEVÉTELÉVEL

A tanulmányban bemutatott közelítő grafikus és matematikai eljárás lehetővé teszi, hogy a szilárd hasznos ásványok kutatására szolgáló fúrásokban felvett gamma-gamma szelvényekből meghatározhassuk a fúrólyuk átmérőjére korrigált térfogatsúlyértékeket.

A mérőszondák hitelesítő méréseiből meghatározott és egy kiválasztott átmérőre normált bázisegyenlet képezi az eljárás alapját. A bázisegyenlet iránytangensértéke (M) és a fúrólyukátmérő között megtalálható lineáris kapcsolat lehetővé teszi, hogy a bázisátmérőnél nagyobb és kisebb átmérőjű etalonsorokban végzett hitelesítő mérések eredményei segítségével a fúrólyukátmérő-korrekció értékeit kiszámíthassuk.

A KRGG–2–120–60 sY típusú szondára bemutatjuk azt az összefüggést, amely segítségével fúrólyukátmérőre korrigált térfogatsúlyértékek határozhatók meg. Az eljárás alkalmazhatóságát gyakorlati példák bemutatásával igazoljuk.

Л. АНДРАШИ

ПРИБЛИЖЕННОЕ ГРАФИЧЕСКОЕ И МАТЕМАТИЧЕСКОЕ РЕШЕНИЕ ДЛЯ ОПРЕДЕЛЕНИЯ ЗНАЧЕНИЙ ОБЪЕМНОГО ВЕСА СУЧЕТОМ ВЛИЯНИЯ СКВАЖИНЫ

Приближающее графическое и математическое решение, представленное в настоящей статье, обеспечивает определение значений объемного веса, скорректированных на диаметр скважины, которые значения получены по гамма-гамма профилям, снятым в скважинах, пробуренных на твердые полезные ископаемые.

Основу метода представляет базисное уравнение, определенное по калибровочным измерениям измерительных зондов, нормированное на выбранный диаметр. Линейная связь между величиной тангенса базисного уравнения (M) и диаметром скважины делает возможным расчет значений коррекции диаметра скважины с помощью эталонных рядов, диаметр которых больше или меньше базисного диаметра.

По зонду KRGG-2-120-60 sY представим зависимость, с помощью которого возможно определение значений объемного веса, скорректированных на диаметр скважины. Возможность применения метода доказано практическими примерами.