If the propagation velocity in the (liquid or gas-filled) pores is v_1 and in the solid matrix it is v_2, let us define the average velocity by the time average formula (WYLLIE et al. 1956)

$$\frac{1}{v_0}=\frac{p}{v_1}+\frac{1-p}{v_2}. \tag{10}$$

Further, define the deviations from v_0 of the velocities of the two phases by

$$\left.\begin{aligned}\frac{1}{v_1}&=\frac{1}{v_0}(1+\varepsilon_1)\\ \frac{1}{v_2}&=\frac{1}{v_0}(1+\varepsilon_2).\end{aligned}\right\} \tag{11}$$

Thus, from the wave propagation point of view the porous material can be considered as a medium with a random velocity distribution

$$\frac{1}{v(\mathbf{x})}=\frac{1}{v_0}[1+\varepsilon(\mathbf{x})], \tag{12}$$

where $\varepsilon(\mathbf{x})$ is a random function of the coordinate $\mathbf{x}$. At any given spatial point

$$\varepsilon(\mathbf{x})=\begin{cases}\varepsilon_1 & \text{with probability } p\\ \varepsilon_2 & \text{with probability } 1\text{-}p.\end{cases} \tag{13}$$

We will also need $\langle\varepsilon\rangle$ and $\langle\varepsilon^2\rangle$. By Eqs. (10), (11) and (13) clearly

$$\langle\varepsilon\rangle=0$$

and, since

$$\varepsilon_1=\frac{v_0}{v_1}-1, \qquad \varepsilon_2=\frac{v_0}{v_2}-1$$

we have

$$\langle\varepsilon^2\rangle=p\varepsilon_1^2+(1-p)\varepsilon_2^2=p(1-p)v_0^2\left(\frac{1}{v_1}-\frac{1}{v_2}\right)^2. \tag{14}$$

Let us define a new, normalized, random function by

$$\mu(\mathbf{x})=\frac{\varepsilon(\mathbf{x})}{\langle\varepsilon^2(\mathbf{x})\rangle^{1/2}} \tag{15}$$

and its autocorrelation function as

$$N(\mathbf{x},\mathbf{x}')=\langle\mu(\mathbf{x})\cdot\mu(\mathbf{x}')\rangle. \tag{16}$$

If the random function $\mu(\mathbf{x})$ (that is $\varepsilon(\mathbf{x})$) is homogeneous and isotropic (cf.e.g. TATARSKI 1961), $N(\mathbf{x},\mathbf{x}')$ only depends on the magnitude of $\mathbf{x}-\mathbf{x}'$ that is,

$$N(\mathbf{x},\mathbf{x}')=N(|\mathbf{x}-\mathbf{x}'|)=N(r)(r=|\mathbf{x}-\mathbf{x}'|). \tag{17}$$

The solution of the random wave equation

$$\Delta u+\frac{\omega^2}{v^2}u=0 \tag{18}$$

(where $v=v(\mathbf{x})$ is the randomly distributed velocity, Eq. (12)), can be derived by KELLER's (1964) perturbation method, see Appendix. It turns out that, providing

$$\langle\varepsilon^2\rangle\ll 1, \tag{19}$$

the effective attenuation coefficient of the porous medium is given by

$$\alpha=\langle\varepsilon^2\rangle k_0^2\int_0^\infty(1-\cos 2k_0 r)N(r)\,dr, \tag{20}$$

where $N(r)$ is the normalized autocorrelation function of $\varepsilon(\mathbf{x})$, and k_0 the average wave number

$$k_0=\frac{\omega}{v_0}=\frac{2\pi f}{v_0}. \tag{21}$$

According to Eq. (20) we have to compute $N(r)$ to obtain an explicit expression for α. Assume that an arbitrary line e be drawn through the porous medium. Points on the line are to be defined by giving their arc length x from an arbitrary origin. Then, for certain values of x, the line will pass through pore spaces, for other values of x the line will pass through the solid matrix. We introduce a function $\mu(x)$ along this line defined as follows: the value of μ is defined as $\mu_1=\varepsilon_1/\langle\varepsilon^2\rangle^{1/2}$ (*cf.* Eqs. 13 and 15) if the line at x passes through a pore space; it is defined as equal to $\mu_2=\varepsilon_2/\langle\varepsilon^2\rangle^{1/2}$ if the line passes through the solid matrix. (This is essentially similar to the way that FARA and SCHEIDEGGER 1961 characterized the statistical geometry of porous rocks.) On the basis of the ergodic hypothesis $\langle\mu(x)\rangle=0$, $\langle\mu^2(x)\rangle=1$ along the line e as well.

By the construction of the function $\mu(x)$ it follows that it will consist of randomly occurring square wave impulses of random width.

First, we show that the number of these square waves within an arbitrary interval $[x_1, x_2]$, $|x_2-x_1|=x$, obeys a Poisson distribution. Indeed, if the function possesses n square waves between x_1 and x_2, this means that the line e intersects exactly n pores within this interval. But this can occur if and only if there are exactly n pores inside the cylinder H of radius a around the axis $\overline{x_1x_2}$.

Because of Eq. (7),

$$p_n=P(n\text{ pores in cylinder }H)=\exp(-\lambda_3 a^2\pi x)\frac{(\lambda_3 a^2\pi x)^n}{n!}, \tag{22}$$

that is, the number of square wave impulses in the random function $\mu(x)$ also obeys a Poisson distribution of parameter

$$\lambda_1=\lambda_3 a^2\pi. \tag{23}$$

The mean width $\bar{\vartheta}$ of these square wave impulses can also be easily determined.

Suppose the expected number of pores in the cylinder H is N, then the porosity $\langle p\rangle$ is given by

$$\langle p\rangle=N\cdot\frac{4a^3\pi}{3}\cdot\frac{1}{a^2\pi x}=\frac{4Na}{3x}. \tag{24}$$

Because of ergodicity, the same porosity should arise when estimated along the axis $\overline{x_1x_2}$ of H, that is,

$$\frac{N\bar{\vartheta}}{x}=\frac{4Na}{3x}$$

and

$$\bar{\vartheta}=\frac{4}{3}a. \tag{25}$$

The autocorrelation function

$$N(x)=\langle\mu(x_1)\cdot\mu(x_2)\rangle(|x_2-x_1|=x)$$

of the Poisson-distributed square wave pulses of random width $\bar{\vartheta}$ is, by the well known CAMPBELL formula of radio physics (see e.g. RYTOV 1966):

$$N(x)=\lambda_1\bar{\vartheta}\exp\left(-x/\bar{\vartheta}\right),$$

that is, by Eqs. (23), (25) and (8)

$$N(x)=\lambda_3 a^2\pi\cdot\frac{4}{3}\exp\left(-x\Big/\frac{4}{3}a\right)=\lambda_3 c\exp\left(-x\Big/\frac{4}{3}a\right). \tag{26}$$

Inserting Eq. (26) into (20) and carrying out the integration we get

$$\alpha=\langle\varepsilon^2\rangle k_0^2\lambda_3 c\frac{4\cdot\left(\frac{4}{3}a\right)^3 k_0^2}{1+4\left(\frac{4}{3}a\right)^2 k_0^2}. \tag{27}$$

In the low frequency limit, providing that

$$4\cdot\left(\frac{4}{3}a\right)^2 k_0^2\ll 1 \tag{28}$$

we obtain

$$\alpha=\text{const}\,\langle\varepsilon^2\rangle\lambda_3 c^2 k_0^4, \tag{29}$$

which is very similar to BELTZER's (1978) result

$$\beta=\text{const}\cdot\omega^4\cdot\lambda\cdot\langle c^2\rangle[v]^{-1}.$$

(The role of the $\langle\varepsilon^2\rangle$ factor in Eq. (29) will be discussed later.)

3. Generalizations for random pore-size distribution

Let us now consider the more general case* when, instead of a single constant radius a, the pores may have different radii $a_1, a_2, \ldots, a_k$ with the respective probabilities

$$p_1, p_2, \ldots, p_k; \quad p_i\geqq 0, \quad \sum_{i=1}^{k} p_i=1.$$

* The transition to continuous pore-size distributions seems to be rather complicated and is postponed to a further study.

The previously discussed event that the interval $[x_1, x_2]$ of the random line e intersects exactly N pores in such a way that it intersects

$$\left.\begin{array}{l} n_1 \text{ pores of radius } a_1 \\ n_2 \text{ pores of radius } a_2 \\ \dots\dots\dots\dots\dots\dots \\ n_k \text{ pores of radius } a_k \end{array}\right\} \quad n_1+n_2+\ldots+n_k=N$$

occurs if and only if:

there are n_1 pores of radius a_1 inside the cylinder H_1 of radius a_1, around axis $[x_1, x_2]$;
there are n_2 pores of radius a_2 inside the cylinder H_2 of radius a_2, around axis $[x_1, x_2]$;
....
there are n_k pores of radius a_k inside the cylinder H_k of radius a_k, around axis $[x_1, x_2]$.

The probability of this event is

$$P(n_1, n_2, \ldots, n_k)=\exp(-p_1\lambda_3 a_1^2\pi x)\cdot\frac{(p_1\lambda_3 a_1^2\pi x)^{n_1}}{(n_1)!}\cdot\ldots\exp(-p_k\lambda_3 a_k^2\pi x)\cdot\frac{(p_k\lambda_3 a_k^2\pi x)^{n_k}}{(n_k)!}=$$

$$=\prod_{i=1}^{k}\exp(-p_i\lambda_3 a_i^2\pi x)\prod_{i=1}^{k}\frac{(p_i\lambda_3 a_i^2\pi x)^{n_i}}{(n_i)!}\,. \tag{30}$$

To find the probability $P(N)$ that there are exactly N intersections in the interval $[x_1, x_2]$ (i.e. that $\mu(x)$ contains N square wave impulses in this interval), we have to sum Eqs. (30) for all $\{n_1, n_2, \ldots, n_k\}$-s, for which $n_1+\ldots+n_k=N$.

Observing that

$$\langle a^2\rangle^N=(p_1a_1^2+\ldots+p_ka_k^2)^N=\sum_{n_1+n_2+\ldots+n_k=N} N!\prod_{i=1}^{k}\frac{(p_ia_i^2)^{n_i}}{(n_i)!}\,,$$

we have

$$\sum_{n_1+n_2+\ldots+n_k=N}\prod_{i=1}^{k}\frac{(p_ia_i^2)_{n_i}}{(n_i)!}=\frac{\langle a_2\rangle^N}{N!} \tag{31}$$

and, by Eq. (30),

$$P(N)=\sum_{n_1+\ldots+n_k=N}P(n_1, n_2, \ldots, n_k)=\exp[-\lambda_3\pi x\langle a^2\rangle]\cdot\frac{(\pi x\lambda_3\langle a^2\rangle)^N}{N!}\,. \tag{32}$$

Thus, the square wave impulses of the function $\mu(x)$ occur again according to a Poisson distribution of density λ_1,

$$\lambda_1=\lambda_3\pi\langle a^2\rangle, \tag{33}$$

(compare with Eq. 23). The average pulse width becomes, in this case, $\bar{\vartheta}=\frac{4}{3}\langle a\rangle$. Proceeding as in the previous section,

$$N(x)=\lambda_3\pi\cdot\frac{4}{3}\langle a^2\rangle\langle a\rangle\exp\left[-x\Big/\frac{4}{3}\langle a\rangle\right] \tag{34}$$

and, for low frequencies, we find that the attenuation coefficient becomes

$$\alpha = \text{const} \cdot \langle \varepsilon^2 \rangle \lambda_3 \langle a^2 \rangle \langle a \rangle^4 \cdot k_0^4, \tag{35}$$

an expression still analogous to BELTZER's formula. Here, as in Eq. (29),

$$\langle \varepsilon^2 \rangle = p(1-p)\left(\frac{1}{v_1} - \frac{1}{v_2}\right)^2 \cdot v_0^2 .$$

The main differences between BELTZER's result and our expressions (29) and (35) are that according to our results the attenuation coefficient also depends on the velocity contrast $(v_1 - v_2)^2$ and has a further porosity dependence of the form $p(1-p)$.

In 1953 AMENT proposed a theoretical expression for the attenuation of sound waves in a suspension:

$$\alpha(\omega) = \frac{\omega^2}{9\eta} \cdot \frac{p(1-p)}{c_0^2} \cdot \frac{(\varrho_1 - \varrho_2)^2}{\varrho_0^2} r^2 \tag{36}$$

where η is the viscosity of the fluid, p porosity, c_0 average velocity, r radius of the solid particles, ϱ_2 and ϱ_1 densities of the fluid and of the solid particles, respectively. ϱ_0 is the average density:

$$\varrho_0 = p\varrho_1 + (1-p)\varrho_2 .$$

In the late fifties, this formula was widely used in geophysical practice (OFFICER 1955, BERZON et al. 1959). Because of the very well established over-all positive correlation between densities and velocities (NAFE and DRAKE 1957, HAMILTON 1970), AMENT'S Eq. (36) suggests a proportionality of the attenuation coefficient to the velocity contrast, in the same way as implied by Eqs. (29) and (35).

The proportionality of α with $p(1-p)$ in Eq. (36) and our (29), (35) has been experimentally confirmed over a wide range of porosities by SHUMWAY (1960) and HAMILTON (1972), for marine sediments. Few systematic studies have been made on the role of the velocity (or density) contrast (see, however, ZEMTSOV (1965) and the references cited in KORVIN 1977 p.29).

4. Concluding remarks, connections with information theory

In the two previous sections it has been shown (Eqs. 29 and 35) that in a randomly porous medium, and for the low-frequency regime, the attenuation coefficient is proportional

- to the density λ_3 of the Poisson distribution of the number of pores;
- to the expression $p(1-p)$, p being average porosity;
- to the velocity contrast of the two phases;
- to higher momenta of the pore-size distribution.

From among these factors, we consider the first two as characteristic to the "randomness" of pore-geometry, at least in a qualitative sense. The $p(1-p)$ factor plays an especially interesting role. Indeed, as we have recently reported (KORVIN 1978), if we consider multiphase materials instead of 2-phase ones, the attenuation coefficient will be proportional to

$$H = \sum_{i=1}^{n} p_i(1-p_i) \tag{37}$$

where p_i is the relative volume ratio (i.e. probability of occurrence at any given point) of the

i-th phase, $p_i \geq 0$, $\Sigma p_i = 1$. The quantity H (termed *heterogeneity factor* in Korvin 1978) has the properties:

$H=0$ if any of the p_i-s is 1, it attains its maximum for the distribution

$$p_1 = p_2 = \ldots = p_n = \frac{1}{n},$$

the maximum being

$$H_{\max} = \frac{n-1}{n}. \tag{38}$$

It is worth-while to compare the heterogeneity factor with the entropy

$$E = -\sum_{i=1}^{n} p_i \log p_i \tag{39}$$

of the probability distribution which, of course, is a more appropriate measure of the randomness of a multiphase material (Byryakovsky 1968). It also holds that $E=0$, if any of the p_i-s is 1; E attains its maximum for the distribution

$$p_1 = p_2 = \ldots = p_n = \frac{1}{n},$$

$$E_{\max} = \log n. \tag{40}$$

Applying the series development

$$-x \log x \approx \frac{1}{n} \log n + \left(x - \frac{1}{n}\right)(\log n - 1) - \frac{\left(x - \frac{1}{n}\right)^2}{2} \cdot n \quad \text{if} \quad \left|x - \frac{1}{n}\right| \ll 1$$

and the identity

$$\sum_{i=1}^{n} \left(p_i - \frac{1}{n}\right)^2 = \frac{n-1}{n} - \sum_{i=1}^{n} p_i(1-p_i),$$

we obtain that, provided

$$\sum_{i=1}^{n} \left(p_i - \frac{1}{n}\right)^2 \ll 1$$

one has

$$E = -\sum_{i=1}^{n} p_i \log p_i \approx \log n - \frac{n}{2} \sum_{i=1}^{n} \left(p_i - \frac{1}{n}\right)^2 = \log n - \frac{1}{2}(n-1) + \frac{n}{2} \sum_{i=1}^{n} p_i(1-p_i),$$

that is, if all the p_i-s are close enough to $\frac{1}{n}$:

$$H = \sum_{i=1}^{n} p_i(1-p_i) = \frac{2}{n} E - \frac{2 \log n}{n} + \frac{n-1}{n}$$

or, by Eqs. (38), (40),

$$H_{\max} - H = \frac{2}{n}(E_{\max} - E). \tag{41}$$

Thus, around the maximum, the heterogeneity factor H behaves similarly to the entropy E.

Figure 1 shows the striking similarity of H/H_{max} and E/E_{max} for $n=2$; for $n=3$ see Fig. 1C of HARRIS and MCCAMMON (1969).

Thus, it is reasonable to assume as a hypothesis that *for multiphase materials the low-frequency attenuation coefficient—although not simply proportional—is certainly positively correlated with the entropy, i.e. the randomness, of the material distribution of the medium.*

In this connection it should be noted that the proportionality factor

$$H=\sum_{i=1}^{n} p_i(1-p_i)$$

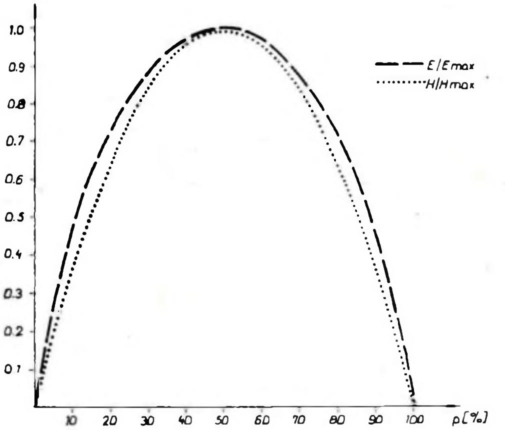

Fig. 1. Relative entropy and relative heterogeneity factor for n=2

1. ábra. Relative entropia és relatív heterogenitási faktor n=2 esetén

Рис. 1. Относительная энтропия и коэффициент мутности для n=2

figuring in the expression of the attenuation coefficient of multiphase materials can also be written as

$$H=1-\sum_{i=1}^{n} p_i^2. \tag{42}$$

The expression

$$\sum_{i=1}^{n} p_i^2$$

was introduced by ONINESCU (1966) as a measure of the *information energy* of the probability distribution $p_1, p_2, \ldots, p_n$. The information energy ranges from $1/n$ (in case of complete indeterminacy) to 1 (if any of the p_i-s is 1); and—as observed by MARCUS (1967, 1971) in his works on mathematical linguistics—it shows the same analogy to the kinetic energy of physical systems as that between the BOLTZMANN entropy and SHANNON's "$-\Sigma p_i \log p_i$". It can be shown that, as in thermodynamics, if the information energy of a system decreases its entropy should increase, and *vice versa*. This, in view of Eq. (42), further corroborates our hypothesis.

It is well-known that the frequency-dependent attenuation and velocity dispersion lead to a distortion of the propagating acoustic pulses. KUZNETSOV et al. (1973) and HOLLIN and

JONES (1977) recently proposed that the correlation between the propagating pulses for the determination of the attenuation characteristics be measured. Theoretically, the propagation of the two-point correlation function (as of any other quadratic quantities) can be described by the BETHE–SALPETER equation (BOURRET 1962) or, alternately, by appropriate transport equations (see e.g. BUGNOLO 1960). In connection with the latter approach FRISCH poses the following problem (1968, p. 145):

"... there are some physical difficulties in the interpretation of the solution, which have not been settled yet. It appears, for example, that in contradistinction to the homogeneous nonrandom case, there is an energy loss, even when the medium is not dissipative".

It seems to us that this problem, together with all the problems prosed in the present paper, will have been solved by following up the pioneering ideas of CASTI and TSE who, in 1972, showed that the KALMAN–BUCY optimal filtering theory and radiative transfer theory, "which from a physical point of view seem to have very little in common, may be brought together by careful examination of their respective initial value formulations" (*op. cit.* p. 42).

In their concluding remarks CASTI and TSE (1972, p. 53) state:

"In conjunction with the active filtering problem, let us mention a radiative transfer function ... this is the absorption function which is defined by means of a conservation law, i.e. it corresponds to the radiative energy which is input to the atmosphere, but which is neither transmitted through nor reflected back out.... In the active filtering case there is reason to suspect that this function may correspond to a loss of inherent information in the known control input due to interaction with the noisy system. If this correspondence can be made precise, it would seem to be possible to establish a conservation of information law for stochastic systems".

Appendix: Perturbation solution to the random wave equation

The main ideas of KELLER's method of stochastic perturbation (KELLER 1964, KARAL and KELLER 1964) are as follows. Suppose the wave u_0 propagating in a space free of inhomogeneities satisfies a linear equation

$$Lu_0=0. \tag{A.1.}$$

Consider now a randomly inhomogeneous space characterized by the operator

$$L-\varepsilon L_1(\gamma)-\varepsilon^2 L_2(\gamma)+0(\varepsilon^3), \tag{A.2.}$$

where ε is a measure of the inhomogeneities of the medium ($|\varepsilon|\ll 1$); $L_1(\gamma)$ and $L_2(\gamma)$ are random operators depending on a random parameter $\gamma\in\Gamma$ of density function $p(\gamma)$. Expectancies with respect to $p(\gamma)$ are denoted by

$$\langle f\rangle=\int_\Gamma f(\gamma)\,p(\gamma)\,\mathrm{d}\gamma.$$

The solution u to the equation

$$[L-\varepsilon L_1(\gamma)-\varepsilon^2 L_2(\gamma)+0(\varepsilon^3)]u=0 \tag{A.3.}$$

is a random function of γ; our aim is to find an equation for the expected wave $\langle u\rangle$. If L^{-1} exists and is bounded, then from Eqs. (A.1.) and (A.3.),

$$u=u_0+\varepsilon L^{-1}(L_1+\varepsilon L_2)u+0(\varepsilon^3). \tag{A.4.}$$

Solving (A.4.) by the successive iteration method, we get

$$u = u_0 + \varepsilon L^{-1} L_1 u_0 + \varepsilon^2 (L^{-1} L_1 L^{-1} L_1 + L^{-1} L_2) u_0 + 0(\varepsilon^3) \tag{A.5.}$$

that is, taking expectancies,

$$\langle u \rangle = u_0 + \varepsilon L^{-1} \langle L_1 \rangle u_0 + \varepsilon^2 L^{-1} (\langle L_1 L^{-1} L_1 \rangle + \langle L_2 \rangle) u_0 + 0(\varepsilon^3). \tag{A.6.}$$

Hence:

$$u_0 = \langle u \rangle - \varepsilon L^{-1} \langle L_1 \rangle u_0 + 0(\varepsilon^2) = \langle u \rangle - \varepsilon L^{-1} \langle L_1 \rangle u + 0(\varepsilon^2), \tag{A.7.}$$

which, on substitution into (A.6.), gives

$$\langle u \rangle = u_0 + \varepsilon L^{-1} \langle L_1 \rangle \langle u \rangle + \varepsilon^2 L^{-1} [\langle L_1 L^{-1} L_1 \rangle - \langle L_1 \rangle L^{-1} \langle L_1 \rangle + \langle L_2 \rangle] \langle u \rangle + 0(\varepsilon^3). \tag{A.8.}$$

Applying L to both sides, dropping the $O(\varepsilon^3)$ term and assuming that $\langle L_1 \rangle = 0$ we finally arrive at

$$(L - \varepsilon^2 \langle L_1 L^{-1} L_1 \rangle - \varepsilon^2 \langle L_2 \rangle) \langle u \rangle = 0 \tag{A.9.}$$

which is an explicit equation for $\langle u \rangle$. Introducing the Green function $G(\mathbf{x}, \mathbf{x}')$ defined as

$$LG(\mathbf{x}, \mathbf{x}') = I \cdot \delta(\mathbf{x} - \mathbf{x}') \tag{A.10.}$$

(where I is the unit operator, δ is Dirac's delta function), we have

$$L^{-1} f = \int G(\mathbf{x}, \mathbf{x}') f(\mathbf{x}')\, d\mathbf{x}'$$

and Eq. (A.9.) becomes

$$L(\mathbf{x}) \langle u(\mathbf{x}) \rangle - \varepsilon^2 \left\langle L_1(\mathbf{x}) \int G(\mathbf{x}, \mathbf{x}') L_1(\mathbf{x}') \langle u(\mathbf{x}') \rangle\, d\mathbf{x}' \right\rangle - \varepsilon^2 \langle L_2(\mathbf{x}) \rangle \langle u(\mathbf{x}) \rangle = 0. \tag{A.11.}$$

Let us now apply the general expression (A.11.) to the random wave equation

$$\Delta u + \frac{\omega^2}{v^2} u = 0 \tag{A.12}$$

where, as in Eq. (12), the random velocity distribution is given by

$$v(\mathbf{x}) = v_0 (1 + \varepsilon(x))^{-1}.$$

If we introduce the average wave number $k_0 = \omega / v_0$ the wave equation becomes

$$\Delta u(\mathbf{x}) + k_0^2 [1 + 2\varepsilon \mu(\mathbf{x}) + \varepsilon^2 \mu^2(\mathbf{x})] u(\mathbf{x}) = 0 \tag{A.13.}$$

where $\varepsilon = \langle \varepsilon^2(\mathbf{x}) \rangle^{1/2}$; the normalized random variable $\mu(\mathbf{x})$ is given by

$$\mu(\mathbf{x}) = \frac{\varepsilon(\mathbf{x})}{\langle \varepsilon^2(\mathbf{x}) \rangle^{1/2}} = \frac{\varepsilon(\mathbf{x})}{\varepsilon}.$$

If Eq. (A.13.) is matched with Eq. (A.3.)

$$\left. \begin{aligned} L &= \Delta + k_0^2 \\ L_1 &= -2k_0^2 \mu(\mathbf{x}) \\ L_2 &= -k_0^2 \mu^2(\mathbf{x}). \end{aligned} \right\} \tag{A.14.}$$

Clearly, $\langle L_1\rangle=0$, $\langle\mu^2(\mathbf{x})\rangle=1$. On imposing the radiation condition, the Green function becomes

$$G(\mathbf{x},\mathbf{x}')=-\frac{\exp[ik_0|\mathbf{x}-\mathbf{x}'|]}{|\mathbf{x}-\mathbf{x}'|} \tag{A.15.}$$

and Eq. (A.11.) will change to

$$(\Delta+k_0^2)\langle u\rangle+\frac{4\varepsilon^2k_0^4}{4\pi}\left\langle\mu(\mathbf{x})\int\frac{\exp[ik_0|\mathbf{x}-\mathbf{x}'|]}{|\mathbf{x}-\mathbf{x}'|}\cdot\mu(\mathbf{x}')\right\rangle\langle u(\mathbf{x}')\rangle\,d\mathbf{x}'+\varepsilon^2k_0^2\langle u\rangle=0. \tag{A.16.}$$

In the case of homogeneous isotropic randomness (TATARSKI 1961) Eq. (A.16.) simplifies to

$$(\Delta+k_0^2+\varepsilon^2k_0^2)\langle u(\mathbf{x})\rangle+\frac{4\varepsilon^2k_0^4}{4\pi}\int\frac{\exp(ik_0r)}{r}N(r)\langle u(\mathbf{x}+\mathbf{r})\rangle\,d\mathbf{r}=0, \tag{A.17.}$$

where $r=|\mathbf{r}|$; $N(r)$ is the autocorrelation function defined by Eqs. (16), (17). Solutions to Eq. (A.17.) will be sought for in the plane-wave form

$$\langle u(\mathbf{x})\rangle=Ae^{i\mathbf{kx}}=\varphi(\mathbf{x}). \tag{A.18.}$$

To compute the volume integral in (A.17.) we first integrate over the spherical surface S of radius r, centred at $\mathbf{x}$. Making use of the mean-value theorem (KELLER 1964)

$$\frac{1}{4\pi r^2}\int_S\varphi(\mathbf{x}+\mathbf{r})\,dS=\frac{\sin kr}{kr}\varphi(\mathbf{x}) \tag{A.19.}$$

which holds for any solution of the wave equation, from Eq. (A.17.) we have

$$\left[\Delta+k_0^2+\varepsilon^2k_0^2+4\varepsilon^2\frac{k_0^4}{k}\int_0^\infty\exp(ik_0r)\sin krN(r)\,dr\right]\varphi(\mathbf{x})=0. \tag{A.20.}$$

Since the plane wave $\varphi(\mathbf{x})$ as defined in Eq. (A.18.) evidently satisfies a wave equation

$$(\Delta+k^2)\,\varphi(\mathbf{x})=0, \tag{A.21.}$$

we obtain, by equating Eqs. (A.20.) and (A.21.), the *dispersion relation*

$$k^2=k_0^2+\varepsilon^2k_0^2+4\varepsilon^2\frac{k_0^4}{k}\int_0^\infty\exp(ik_0r)\sin krN(r)\,dr \tag{A.22.}$$

whose solution k is the *effective wave number* expressing the *global effect* of the velocity inhomogeneities. If (A.22.) is solved in powers of ε and the $0(|\varepsilon|^3)$ terms are dropped (for details, see KORVIN 1977), the imaginary part of k, i.e. the attenuation coefficient α, becomes

$$\alpha=\varepsilon^2k_0^2\int_0^\infty(1-\cos 2k_0r)N(r)\,dr \tag{A.23.}$$

which is the same expression as Eq. (20) of the main part of this paper.

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KORVIN GÁBOR

A VÉLETLEN POROZITÁS HATÁSA A RUGALMAS HULLÁMOK ELNYELŐDÉSÉRE

A dolgozat a porózus kőzegben terjedő rugalmas hullámok elnyelődését vizsgálja a véletlen pórus-elhelyezkedés, póruseloszlás és pórusszám függvényében. A véletlen hullám-egyenlet perturbációs megoldásával új levezetést nyerünk Beltzer nemrég közölt eredményére. Információelméleti megfontolások azt a hipotézist sugallják, hogy *többkomponensű közegek alacsony-frekvenciás elnyelődési együtthatója pozitiv korrelációt mutat a közeg anyageloszlásának entropiájával, vagyis véletlenségével.*

Г. КОРВИН

ВЛИЯНИЕ СЛУЧАЙНОЙ ПОРИСТОСТИ НА ПОГЛАЩЕНИЕ УПРУГИХ ВОЛН

Статья исследует поглащение упругих волн распостраняющихся в пористой среде, в зависимости от случайного расположения, распределения и количества пор. Методом возмущения случайного волнового уравления получим новый вывод на недавно сообщенный результат Бельцера. Рассуждения по теории информации подсказывают гипотезу, что *коэффициент низкочастотного поглащения многокомпонентных сред положительно коррелирует с энтропией, т. е. случайностью материального распределения среды.*