Magyar Állami Eötvös Loránd Geofizikai Intézet
GEOFIZIKAI KÖZLEMÉNYEK
XX. kötet, 1-2. szám.

ANALYSIS OF THE TER-DIURNIAL TIDAL GRAVITY VARIATIONS IN TIHANY

P. VARGA*

The gravity potential of the Earth's tide is described by the formula

$$W = \sum_{n=2}^{\infty} W_n = \sum_{n=2}^{\infty} \frac{f \cdot a^n \cdot m}{R^{n+1}} P_n(\Theta), \tag{1}$$

where W_n is the potential of a tide of n-th order, a is the radius of the Earth. R is the distance between the centres of the Earth, resp. of the tide-generating mass, f is the gravitational constant, m is the tide-generating mass. Term $n=2$ of this formula and of its derivates give useful information about the Earth's interior. It will be shown that term $n=3$, i.e. the ter-diurnal wave, also renders good services in searching our planet. From these waves the one, called M_3 by DARWIN, shows the greatest amplitude, though it does not exceed, on medium latitudes, 0,3 – 0,4 microgals.

The eight-hour wave was first proved by BARSENKOV (1967), based on long recordings of the Talgar observatory. MELCHIOR and VENEDIKOV (1968) established amplitude-quotients similar to those of theoretical Earth models, for several stations and long recordings.

Wave M_3 is in connection with term $n=3$. Its amplitude-quotient is a function of another combination of Love-numbers h_n and k_n than in the case of term $n=2$, usually examined.

Taking an arbitrary term n, differentiating (1), the gravity effect on the surface of an absolutely rigid Earth (PARIYSKIY, 1963) is

$$-\Delta g_0 = \sum_{n=2}^{\infty} \frac{n \cdot W_n}{a} \tag{2}$$

Considering an elastic Earth, the tidal gravity effect is

$$-\Delta g = \frac{\partial \sum_{n=2}^{\infty} W_n}{\partial r} + \frac{\partial \sum_{n=2}^{\infty} k_n(r) W_n}{\partial r} + \sum_{n=2}^{\infty} \frac{n \cdot g \cdot \zeta}{a}, \tag{3}$$

Manuscript received: 2, 11, 1970

* Hungarian Geophysical Institute R. E., Budapest

where r is the distance of the Earth's centre from the point studied. The first term on the right side is the gravity difference caused by the tidal force. The second term is a secondary Δg change originating from the deformation of the elastic Earth. The third term is brought about because of change ξ in the level of the site of recording.

It is known that

$$\frac{\partial W_n}{\partial r} = \frac{n \cdot W_n}{r}$$

and

$$\frac{\partial k_n(r)}{\partial r} = \frac{\partial k_n \frac{a^{2n+1}}{r^{2n+1}}}{\partial r} = -k_n \cdot (2n+1) \cdot \frac{a^{2n+1}}{r^{2n+2}} .$$

Thus, if $a = r$, then

$$\frac{\partial k_n(r)}{\partial r} = -(2n+1) \cdot \frac{k_n}{r}$$

and the height of the statical tide is

$$\zeta_n = h_n \frac{W_n}{g} .$$

Hence, (3) can be transformed to the following form:

$$-\Delta g = \left(1 - \frac{n+1}{n} \cdot k_n + \frac{2}{n} \cdot h_n\right) \cdot \sum_{n=2}^{\infty} \frac{n \cdot W_n}{a} , \tag{4}$$

and for δ_n one obtains:

$$\delta_n = \frac{\Delta g}{\Delta g_0} = 1 - k_n \frac{n+1}{n} + \frac{2}{n} \cdot h_n$$

In case $n = 2$ it means:

$$\delta_2 = 1 - 3/2 k_2 + h_2 ,$$

while in case $n = 3$ it means:

$$\delta_3 = 1 - 4/3 k_3 + 2/3 h_3 .$$

Further, values h and k and their ratio obviously change, for the density distribution of the Earth's interior will get different weights in each of the cases. Namely, in case of ter-diurnal waves, surface-nearer shells get greater weight, as noticed earlier by Melchior (1950), too, who stated at the same time, that density in the Earth's interior (ϱ) is a function of r.

Table I shows, for some Earth-models, the parameters characteristic for tides of second and third order. It seems that examination of third order tide offers information about the whole Earth, somewhat independent from those obtained from second order tides. As a matter of fact, the second order tide gives more accurate values, still the third order one should not be neglected either, for it helps in studying function $\varrho(h)$.

I

	Mo'odenskiy—Kramer (1961)	Takeuchi (1962)	Longman (1963)
n = 2			
h_2	0.617	0.592	0.612
k_2	0.302	0.280	0.302
δ_2	1.164	1.172	1.159
k_2/h_2	0.489	0.473	0.493
n = 3			
h_3	0.294	0.274	0.290
k_3	0.096	0.083	0.093
δ_3	1.068	1.072	1.069
k_3/h_3	0.327	0.303	0.321

Based on the aforementioned considerations, the amplitude-quotient and phase-difference of wave M_3 were determined through a 23 monthly recording in Tihany (22, 2, 1968 — 25, 1, 1970). First the hourly recorded data were treated with interpolated sensitivity-values. Then, without any filtering whatever, the suppression of other than ter-diurnal waves followed, with Fourier transforms (Varga, 1970) on the frequency of wave M_3. Thereafter δ_{M3} has been determined with regard to eventual effects from nearby waves. Theoretical values were taken from Doodson (1922) and Pariyskiy (1961).

Consideration was given to the possible error of the value determined (Table II), the actual value of the error has been established from an examination of the noise level. It became obvious that δ_3 is as slighter than δ_2 as theoretically predicted, namely, the δ average of the greatest waves belonging to $n=2$ (K_1 excluded) is 1,1579.

II

Wave	°/h	Ampl. observed	Ampl. calc.*	$\delta \pm \Delta\delta$
M_3	43.47616	0.5467 μgal	0.4940 μgal	1,107 ± 0,078

* Nearby waves considered.

REFERENCES

BARSENKOV, S. N., 1967: Vychislenie prilivov tret'ego poryadka . . . Fizika Zemli, 15.

DOODSON A. T., 1922: The Harmonic Development of the Tide-generating Potential. Proc. Roy. Soc. A. 100.

LONGMAN, I. M., 1963: A Green's Function for Determining the Deformation of the Earth under Surface Mass Loads. Journal of Geophysical Research, 68. 2.

MELCHIOR P., 1950: Sur l'influence de la loi de repartition des densités á l'intérieur de la terre . . . . Geofisica Pura e Applicata. XVI. 3—4.

MELCHIOR P., VENEDIKOV A., 1968: Derivation of the Wave M_3 (8^h279) from the Periodic Tidal Deformations of the Earth. Physics of the Earth and Planetary Interiors 1. 6.

MOLODENSKIY M. S., KRAMER, M. V., 1961: Zemnye prilivy i nutatsiya Zemli, Moszkva

PARIYSKIY, N. N., 1961: Zamechaniya k vichisleniyu teoreticheskikh znacheniy . . . Izuchenie Zemnyh Prilivov 2.

PARIYSKIY, N. N., 1963: Zemnye prilivy i vnutrenneye stroenie Zemli. Izv. AN SSSR, seriya geofizicheskaya, 1963, 3.

TAKEUCHI, H., SAITO, M., KOBAYASHI N., 1962: Statical Deformations and Free Oscillations of a Model Earth. Journal of Geophysical Research 67. 4.

VARGA, P., 1970: Fourier-Transforms of the Tidal Variations in the Intensity of Gravity. Geofizikai Közlemények, XIX. 3—4., Budapest