Magyar Állami Eötvös Lóránd Geofizikai Intézet
GEOFIZIKAI KÖZLEMÉNYEK
XX. kötet, 1—2. szám

INVESTIGATION OF INTERPOLATION PROCEDURES

T. BODOKY*

One of the basic operations in the digital processing of seismic data is normal correction. In the normal correction of seismic channels, a frequently repeated application of one of the interpolation procedures is necessary. Since several different interpolation procedures may be used in order to select the most suitable one for the problem given, the "goodness" of the individual procedures, serving as a base of comparison must be determined.

By the interpolation procedures, not the required values themselves, but their more or less good approximations are furnished, i.e. they are burdened with errors. As a measure of the "goodness" of the individual procedures, it is advisable to choose just their so-called interpolation errors.

The error of a given procedure can be characterized by the first absolute momentum of the error, giving the average size of the error (for this purpose, not the expectable value of the error is used, since a seismic channel can be divided into trigonometric components; for the latters, however, the expectable interpolation error is, on account of the symmetry of positive and negative sections, always zero). In order to accept a procedure as suitable, this value has to remain under a certain limit. Its determination is best done in a frequency-dependent form:

Be $\varkappa(f)$ the first absolute momentum of the error of a certain interpolation procedure. For the determination of $\varkappa(f)$, let us consider the digitally sampled form of a sin t function with a given frequency f. Be the sampling interval τ — the place of an arbitrary sample t_0. Let us calculate, with a given interpolation procedure, the value belonging to the place t_0+t, lying between the places t_0 and $t_0+\tau$ (if $0 \leq t \leq \tau$). Let us denote this value, burdened with an interpolation error, by $g(t_0, t)$ (Fig. 1) and the interpolation error by h. Then,

$$h = \sin 2\pi f(t_0+t) - g(t_0, t).$$

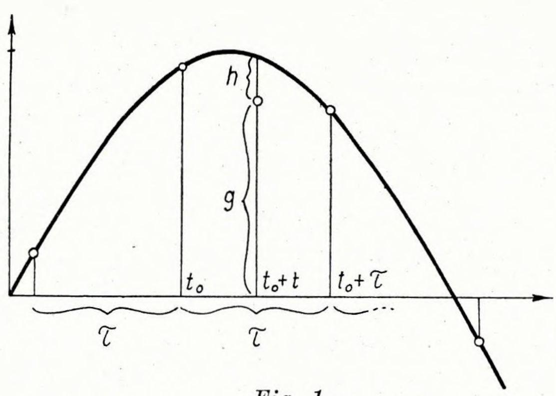

Fig. 1
1. ábra
Рис. 1

* Manuscript received: 17. 8. 1970.
Roland Eötvös Geophysical Institute, Budapest.

In case a certain procedure, the value of h depends on the frequency f, on the sampling interval τ and on the values of t_0 and t. Consequently the interpolation error is a function of a form of $h(f, \tau, t_0, t)$.

In actual data-processing, f and τ are fixed. The choice of t_0 and t is, on the other hand, at random; i.e. these are probability variables. As to their distribution, the following can be said:

t_0 can take any value in the interval $\left(0, \frac{1}{2f} = \frac{T}{2}\right)$ with equal probability.

t can take any value in the interval $(0, \tau)$ with equal probability as a consequence of its choice.

This means that the density functions of both variables are constant, in the interval mentioned, otherwise zero. Considering this, the first absolute momentum, i.e. the expected absolute value of the error is:

$$\alpha(f, \tau) = \int_0^{\frac{T}{2}} \int_0^{\tau} | h(f, \tau, t_0, t) | \frac{1}{\tau} \frac{2}{T} \mathrm{d}t \, \mathrm{d}t_0 .$$

In order to simplify the calculations it is advisable to introduce instead of τ, the dimensionless variable $\tau \cdot f = \nu$, and to choose 1 cps as f. Then we have a function $\alpha(\nu)$, from which, by fixing the value of τ, function $\alpha(f)$, corresponding to any value of τ, can easily be obtained.

In order to illustrate the above-said, the $\alpha(f)$ functions of the stepwise interpolation and linear interpolation have been calculated.

Stepwise interpolation. To the place to be interpolated, always the value of the immediately preceding sampling point is ordered by this procedure (Fig. 2):

$$h = \sin 2\pi(t_0 + t) - \sin 2\pi t_0$$
$$\tau \cdot f = \nu \quad f = 1.$$

As a result of the calculations, the $\alpha(f)$ function for $\alpha(\nu)$, resp. $\tau = 2$ msec is presented in Fig. 3. The percentage values marked on the vertical axis represent the percentage as compared to the amplitude of the original function.

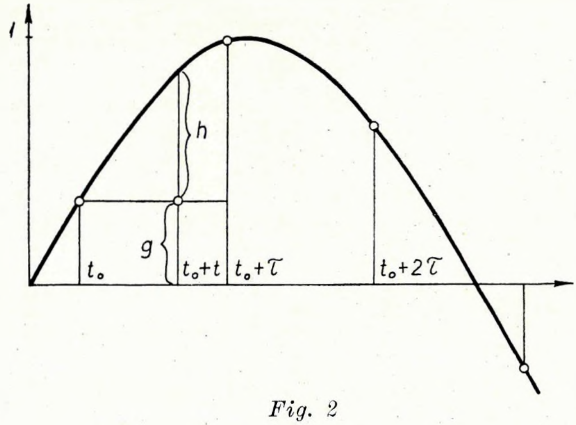

Fig. 2
2. ábra
Рис. 2

Linear interpolation. This orders to the place to be interpolated the value calculated from the value of the two adjacent sampling points in proportion of the distance between the place to be interpolated and the sampling points (Fig. 4):

$$f \cdot \tau = \nu \quad f = 1$$

$$h = \sin 2\pi(t_0 + t) - \frac{t}{\nu} \sin 2\pi(t_0 + \nu) - \frac{\nu - t}{\nu} \sin 2 t_0 .$$

According to the calculations, the $\alpha(f)$ functions for $\alpha(\nu)$, resp. $\tau = 2$ msec, are shown in Fig. 5. In Figs. 6 and 7, two examples are presented for the illustration of the calculation results.

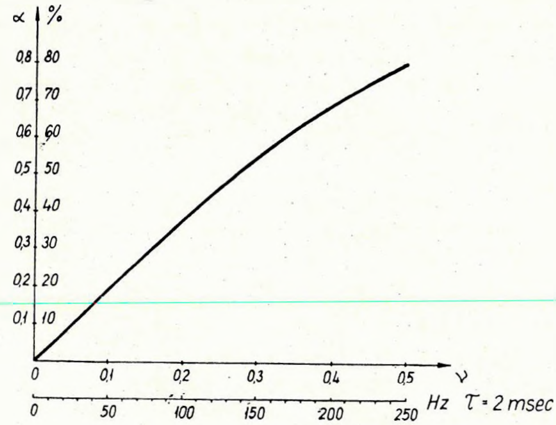

Fig. 3
3. ábra
Рис. 3

In Fig. 6, a sine curve of 28 cps is shown, sampled by 2 milliseconds. This series of data (A) must be expanded in a ratio of 4:5, i.e. the curve values must be read off by 1,6 milliseconds. These values are interpolated from the values of the A series of data, first by the stepwise method. Thus, the data series B, then C, with linear method, will be obtained. For the sake of comparability, the original curve is superposed to all the three data series.

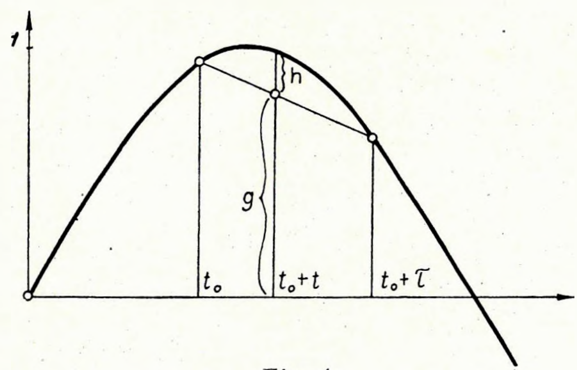

Fig. 4
4. ábra
Рис. 4

In Fig. 7, the same is repeated with a sine curve of 70 cps.

It is visible that, in Fig. 6, the B series of data shows smaller variations in comparison to the actual values, while practically no deviations are present in the C series of data. This corresponds to that result of Figs. 3 and 5, that the average value of the error of the first procedure, at a frequency of 28 cps and a sampling interval of 2 msec, is about 10% of the amplitude of the curve sampled, while the same value is about only 1,5% with the second procedure.

By the deviations of the data series of Fig. 7 the expectable errors of 27, resp. 7% are similarly well reflected.

*

Summing up the results of our calculations, the following conclusion can be drawn:

In the course of seismic data-processing, the frequency range allowed by the Shannon-theorem cannot be entirely exploited, and a narrower (eventually much narrower) one is utilized, the upper limit of which is a function of the maximum error predetermined and of the interpolation procedure applied.

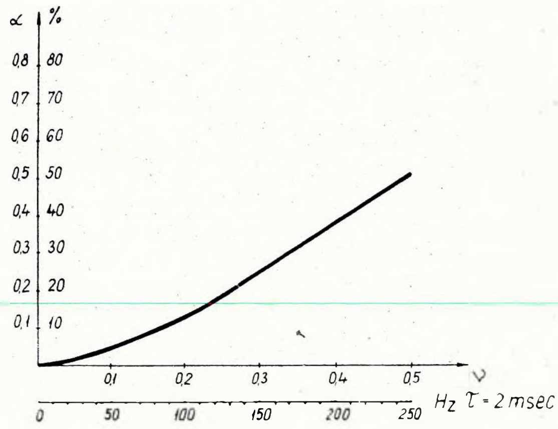

Fig. 5
5. ábra
Рис. 5

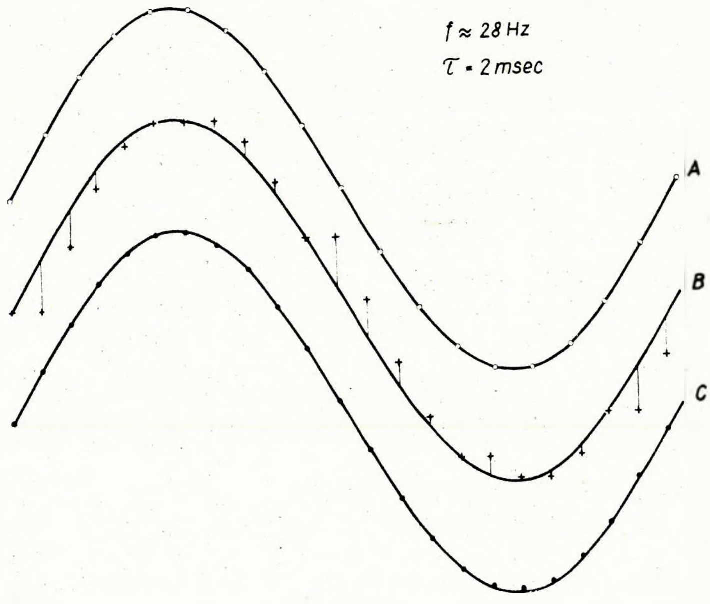

Fig. 6
6. ábra
Рис. 6

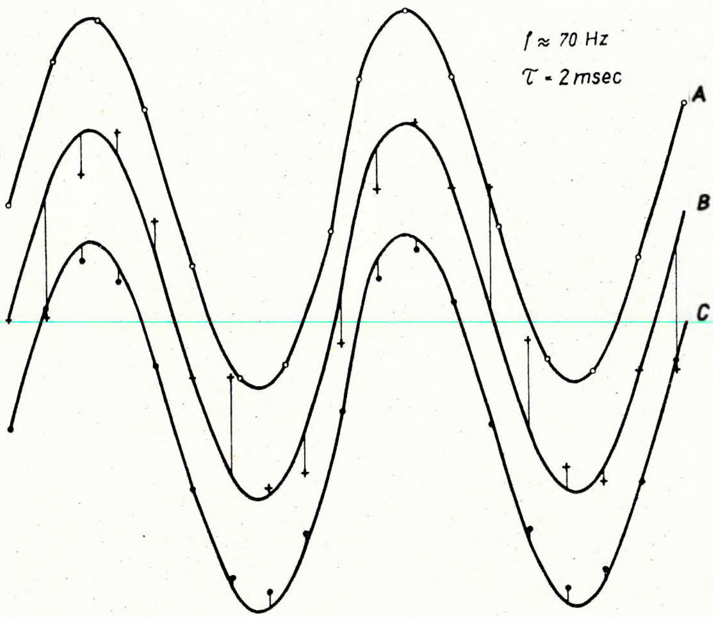

Fig. 7
7. ábra
Рис. 7

It is planned to extend the calculations, in the future, to other interpolation procedures used or usable in seismic data-processing, paying attention to the economic side of each procedure, too.

BODOKY TAMÁS

INTERPOLÁCIÓS ELJÁRÁSOK VIZSGÁLATA

A szeizmikus adatok digitális feldolgozásának egyik alapművelete a normálkorrekció. A cikkben a normálkorrekció egyik sokszor ismételt lépését, az interpolációt vizsgáljuk. Az egyes interpolációs eljárások jellemzésére az interpolációs hiba első abszolút momentumát használjuk. Ezt az f frekvencia és a τ mintavételi távolság szorzatából képzett dimenziótlan ν változó függvényében számítjuk ki. A számolást két egyszerű interpolációs eljárásra végeztük el.

A bemutatott eredményekből kitűnik, hogy a felső határfrekvencia megengedhető értéke függ az alkalmazni kívánt interpolációs eljárástól és az elméleti értéknél sokkal kisebb lehet (Shannon-tétel).

Т. БОДОКИ

ИССЛЕДОВАНИЕ МЕТОДОВ ИНТЕРПОЛЯЦИИ

Одной из основных операций по цифровой обработке сейсмических данных является введение кинематической поправки. В статье рассматривается один из часто повторяющихся шагов по введению этой поправки — интерполяция. Для характеристики отдельных методов интерполяции применяется первый абсолютный момент ошибки интерполяции. Он вычисляется как функция безразмерной переменной ν, образованной из произведения частоты f и шага квантования τ. Вычисление было выполнено для двух простых методов интерполяции.

Из приведенных результатов видно, что допустимое значение верхней предельной частоты зависит от применяемого метода интерполяции и может быть значительно ниже теоретической величины (положение Шаннона).