Simulation Studies on Various Tuning Methods for Convergence Stabilization in a Novel Approach of Model Reference Adaptive Control Based on Robust Fixed Point Transformations

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Abstract: The concept of "*Artificial Intelligence (AI)*" also contains the adaptive controllers that are able to observe the behavior of the *a priori only insufficiently known physical systems* under their control, and automatically can adjust themselves in order to achieve *precise control*. Regarding their implementations, certain approaches use the rather "conventional" means of AI as rule bases, fuzzy, neural or neurofuzzy systems, others can more strictly utilize the specialties of the available analytical models. In this paper the behavior of a novel version of the "*Model Reference Adaptive Controllers*" is investigated. In contrast to the traditional approaches the design of these controllers does not need the use of the difficult technique of Lyapunov's "direct" method. Instead of the use of Lyapunov functions that can guarantee global asymptotic stability it applies "*Robust Fixed Point Transformations*" that work with a local, bounded basin of convergence of the iteration that converges to the solution of the control task. The method applies only three control parameters that in the most of the cases can be fixed. It is shown that by properly tuning only one of the three parameters the convergence of the controller can be stabilized. The theory does not uniquely define the details of this tuning in which we have a great freedom. The operation of various tuning strategies were investigated for the adaptive control for two interesting paradigms: an underactuated mechanical system and an other mechanical system that contains a dynamically coupled internal degree of freedom neither observed nor directly manipulated by the controller.

Keywords: Model Reference Adaptive Control, Lyapunov's Direct Method, Robust Fixed Point Transformations, Contractive Mapping, Cauchy Sequence

1. Introduction

The subject area of Artificial Intelligence includes the set of adaptive controllers. These systems are also intelligent in the sense that they observe the behavior of the *a priori only insufficiently known systems* under their control and automatically adjust themselves in order to achieve *precise control* in spite of the missing *a priori* information. Regarding their implementations, certain approaches use the rather "conventional" means of AI as rule bases, fuzzy, neural or neurofuzzy systems. For instance, the mathematical model of the turbo jet engines (e.g. [1]) allows a kind of "situational control" [2] in which only typical control situations are of interest that can be arranged in a hierarchical structure. The method can be extended for dealing with large scale systems, too [3]. Other implementations can more strictly utilize the specialties of the available analytical models. For realization of *adaptive control of dynamical systems* generally two "traditional" groups of problem tackling exist. One of them uses the dynamical model of the system to be controlled with imprecise model parameters that are precisely tuned by observing its state propagation. This set can be referred to as the group of *"model parameter adaptive controllers"*. Classical representatives of these groups are the "*Adaptive Inverse Dynamics Controllers (AID)*" and the "*Adaptive Slotine–Li Controller (ASLC)*" that use formally exact analytical models, but other types of models based on universal approximators can also be designed in this group. These controllers work well only if it is guaranteed that the observed behavior of the system can be attributed exceptionally to its own dynamics, therefore the presence of unknown external disturbances and dynamically coupled subsystems not involved in the initial model can deceive their model–based tuning process. The "fundamental" significance of this classical approach is well substantiated by the homepage of the *"Nonlinear Systems Laboratory"* of the *"Massachusetts Institute of Technology"* [4] that, according to the *"U.S.News & World Report"* [5], is the World's leading educational institution in the "Engineering and IT" category. This home pages recommends only two fundamental books from 1986 [6] and 1991 [7] and about 90 journal papers written in the era from 1991 to 2010.

The members of the other traditional approach also use imprecise initial dynamic models but instead trying to tune the models' parameters they tune some controllers' parameters. Such kind of controllers can be referred to as *"signal adaptive controllers"* since their tuning processes directly influence the actions of the drives/actuators that realize the control. The "*Model Reference Adaptive Control (MRAC)*" is a typical representative of this group. It is a popular approach from the early nineties to our days (e.g. [8], [9], [10], [11]). The essence of the idea of the MRAC is the transformation of the actual system under control into a well behaving reference system (the *"reference model"*) for which simple controllers can be designed. In the practice the reference model used to be stable linear system of constant coefficients, but in principle it can be any type of prescribed "nominal" system having some "decent behavior". In [9] e.g. C. Nguyen presented the implementation of a joint-space adaptive control scheme that was used for the control of a non-compliant motion of a Stewart platform-based manipulator that was used in the Hardware Real-Time Emulator developed at Goddard Space Flight Center to emulate space operations. In [10]

Somló, Lantos, and Cát suggested and investigated via simulations a local, robust MRAC axis control for robots. The method is also attracting in the control of teleoperation systems [11].

The above mentioned examples of MRAC controllers as well as other appearances of the same idea in the mainstream of control literature applied Lyapunov's "direct method" that originally was elaborated for the investigation of the stability of dynamical systems in his PhD dissertation in 1892 [12], [13]. This method is quite ingenious because it allows to guarantee the stability of the motion of the controlled systems —either global or local, common, exponential or asymptotic stability— on the basis of relatively simple estimations without needing to obtain and study the solutions of the equations of motion. (It is well known that the majority of the practically important problems do not have analytical solutions in closed form, while the numerical solutions are normally valid only for the limited time-span of investigations and without deeper mathematical background their results cannot be extrapolated.) However, in spite of the essential conceptional simplicity of the Lyapunov function technique its practical use is rather an "art" than a simple procedure that could easily be automated. Finding the appropriate Lyapunov function candidate and making the proper mathematical estimations required for the proof of convergence need great mathematical skills and practices, and these difficult proofs normally take pages of papers and generate complicated, nontrivial restrictions to be met. For example in [14] 10 pages of the 14 pages long paper is consumed up with the description of necessary special assumptions and the intricate proof of convergence with a Lyapunov function consisting of 4 components. Such mathematical complications made it reasonable to seek some alternative approach to the realization of the idea of the adaptive controllers. For highlighting these difficulties two, relatively simple examples will be analyzed in the next subsection.

1.1. The Adaptive Slotine–Li Controller

The here presented observations were detailed in [15]. This control approach utilizes subtle details of the Euler–Lagrange equations of motion, namely that the terms quadratic in the generalized velocity components are not independent of the inertia matrix: they can be deduced from the inertia matrix, and according to their special position in the equations of motion they can be symmetrized. In this approach the exerted generalized torque/force components are constructed by the use of the actual model as follows:

$$\begin{gathered} Q = \hat{H}(q)\dot{v} + \hat{C}(q,\dot{q})v + \hat{g} + K_D r, e := q^N - q, v := \dot{q}^N + \Lambda e, \\ r := \dot{e} + \Lambda e, \tilde{p} := \hat{p} - p, C_{ij} = \tfrac{1}{2}\sum_z \dot{q}_z \left(-\tfrac{\partial \hat{H}_{zj}}{\partial q_i} + \tfrac{\partial \hat{H}_{ij}}{\partial q_z} + \tfrac{\partial \hat{H}_{iz}}{\partial q_j} \right) \\ Q = Y(q,\dot{q},v,\dot{v})\hat{p} + K_D r \end{gathered} \tag{1}$$

in which q^N and q denote the generalized co–ordinates of the *nominal* and the *actual* motion, K_D and Λ are symmetric positive definite matrices, matrices $\hat{H}$, $\hat{C}$, and $\hat{g}$ are the actual models of the system's inertia matrix, the Coriolis, and the gravitational terms,

respectively. The possession of the exact form of the dynamical model makes it possible to linearly separate the system's dynamic parameters (the array p in the expression of the physically interpreted *generalized forces* Q) by the use of matrix Y that exclusively consists of precisely known kinematical data. The Lyapunov function of this method is $V = r^T Hr + \tilde{p}^T \Gamma \tilde{p}$, with *positive definite symmetric matrix* Γ. *For guaranteeing asymptotic stability of the control negative derivative is needed for the Lyapunov function.* For this purpose the *skew symmetry* of the matrix C_{ij} and the parameter tuning rule $\dot{\hat{p}} = \Gamma^{-1} Y^T r$ are utilized. *This method has the following difficulties. For a complex dynamic system the array* p *may have many, say* m *elements, to which a matrix* Γ *of size* $m \times m$ *containing* $m + (m^2 - m)/2$ *independent, arbitrary control parameters belongs. These parameters cannot optimally be set, they are present in the tuning rule and make the process of tuning clumsy. Unknown perturbations and coupled, not modeled dynamics can deceive the tuning process. The control method itself requires the use of the complicated analytical expressions giving the arrays* $\hat{H}$, $\hat{C}$, *and* $\hat{g}$. The next example is a simple version of the traditional MRAC controllers.

1.2. A Possible MRAC Controller Designed by the Use of a Lyapunov Function

The here presented realization of the traditional MRAC design was studied in details in [16]. The essence of the idea of the *MRAC* is the transformation of the actual system under control into a well behaving reference system (reference model) for which simple controllers can be designed. In the practice the *reference model* used to be stable linear system of constant coefficients. To achieve this simple behavior normally special adaptive loops have to be developed. For comparison we choose a relatively simple implementation containing integrated feedback in the tracking error. Let the tracking error be denoted as $e := q^N - q$ and let $\xi(t) := \int_0^t e(\tau) d\tau$ (q^N denotes the *nominal*, q stands for the actual *generalized coordinates*). The kinematically prescribed trajectory tracking can be defined by the positive definite matrix Λ and the "error metrics" of the VS/SM controllers as $S := \left(\frac{d}{dt} + \Lambda\right)^3 \xi(t)$ that is required to be zero. If this prescription is realized the integrated tracking error has to vanish exponentially. (According to Barbalat's lemma [7] for uniformly continuous error e it also means that $e \to 0$. Uniform continuity is not an extreme restriction for smooth dynamical systems and uniformly continuous nominal trajectories.) The result is a "*desired 2nd derivative*" $\ddot{q}^D := \ddot{q}^N + \Lambda^3 \xi + 3\Lambda^2 e + 3\Lambda \dot{e}$. Let the reference model consist of two symmetric positive definite constant matrices M^{Ref} and B^{Ref} as $M^{Ref} \ddot{q}^D + B^{Ref} \dot{q} = Q^{Ref}$ where Q^{Ref} corresponds to the force/torque need for the reference model in the actual state of the system defined by the q, and $\dot{q}$ values. Let $H(q)\ddot{q} + h(q,\dot{q}) = Q$ be the actual system's equation of motion. By "copying" the idea of the *Adaptive Inverse Dynamics Controller* let the exerted force/torque be $M^{Ref} \ddot{q}^D + B^{Ref} \dot{q} + D = H(q)\ddot{q} + h(q,\dot{q}) = Q$ in which D corresponds to an additive force to be determined by the MRAC controller. Via subtracting $M^{Ref} \ddot{q}$ from both sides we can express the known difference of the desired and actual joint accelerations as $\ddot{q}^D - \ddot{q} = {M^{Ref}}^{-1} \left[\left(H(q) - M^{Ref} \right) \ddot{q} + h(q,\dot{q}) - B^{Ref} \dot{q} - D \right]$. By the introduction of

the arrays $x := \left[\xi^T,e^T,\dot{e}^T\right]^T$ and $\dot{x} = \left[e^T,\dot{e}^T,\ddot{e}^T\right]^T$ the following equation of motion holds:

$$\dot{x} = Ax + \Phi \quad \texttt{with} \quad A := \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ -\Lambda^3 & -3\Lambda^2 & -3\Lambda \end{bmatrix} \tag{2}$$

and

$$\Phi := \left[\ 0^T,0^T,[M^{Ref^{-1}}\left(\left(H - M^{Ref}\right)\ddot{q} + h - B^{Ref}\dot{q} - D\right)]^T\ \right]^T. \tag{3}$$

in which it can also be written that $\Phi = [0^T,0^T,(\ddot{q}^D - \ddot{q})^T]^T$. With a positive definite matrix P the Lyapunov function $V := x^T P x$ can be introduced with the desired negative time derivative $\dot{V} = x^T\left(A^T P + PA\right)x + 2x^T P\Phi < 0$. By solving the Lyapunov equation $A^T P + PA = -R$ with a *prescribed positive definite symmetric matrix* R appropriate P can be obtained. So the term quadratic in x can be made negative. Therefore, it is sufficient to guarantee the non–positive nature of the remaining one, $x^T P\Phi$. Since $\ddot{q}^D - \ddot{q}$ is known (measurable) the remaining term consists of the sum of *known* and *unknown* parts as

$$\begin{gathered} z_{meas} := x^T P\Phi = x^T P(\ddot{q}^D - \ddot{q}) = \\ x^T P\left[0^T;0^T;M^{Ref^{-1}}\right]\left\{\left(H - M^{Ref}\right)\ddot{q} + h - B^{Ref}\dot{q}\right\} - \\ -x^T P\left[0^T;0^T;M^{Ref^{-1}}\right]D < 0 \end{gathered} \tag{4}$$

in which the 2nd row contains the *unknown part* denoted by u in the sequel, while the *known part* in the 3rd row shortly can be denoted as a product as $w^T D$ with $w^T := x^T P\left[0^T;0^T;M^{Ref^{-1}}\right]$. Let us seek D in the form of $\alpha(t)w$! Then the condition $z_{meas} = u - \alpha(t)w^T w < 0$ should be met. Since $w^T w \geq 0$, in the possession of z_{meas} we have idea if $\alpha(t)$ must be increased or decreased. Let us apply a tuning rule with $\kappa > 0$ as follows: $\dot{\alpha} = \kappa z_{meas}$. With properly great κ and P this tuning can soon lead to decreasing Lyapunov function, i.e. to stable control. *This method has the following weak points: the additional control force D is directly varies with the controlled quantity α. This requires slow, cautious tuning. The tuning rule again contains the numerous arbitrary elements of P (inherited from matrix R) that are not optimally chosen.* Its advantage is that this controller (in contrast to that developed by Slotine & Li) already is able to compensate the effects of unknown external disturbances and coupled, not modeled dynamics

The use of the Lyapunov function technique was found to be avoidable by the application of "*Robust Fixed Point Transformations (RFPT)*" in the design of MRAC controllers not only for "*Single Input – Single Output (SISO)*" systems [17] but for "*Multiple Input – Multiple Output (MIMO)*" systems in [18]. This new adaptive approach in principle can compensate the effects of not modeled coupled dynamics and persistent external

disturbances on which we can obtain information only by observing the motion of the controlled system.

In comparison with Lyapunov's "direct method" this novel approach has only one weak point: its stability is based on a convergent tuning process having only a local, bounded basin of attraction. Therefore, no global stability can be guaranteed by the new method, and its sensitivity to external disturbances and coupled dynamics can be studied via simulations. (However, if numerical details are also needed then the same numerical analysis must be applied in the approaches using some Lyapunov function, since it can guarantee global asymptotic stability without revealing easily interpretable numerical details.) Whenever the external disturbances and the coupled dynamic subsystems show strong time–varying nature this deficiency may be significant.

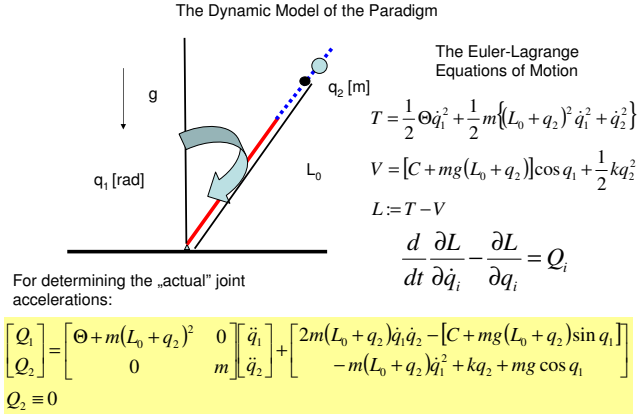

Figure 1: The model of the pendulum with uncertain mass center point: the rotary axle q_1 is directly controlled while the mass m can freely move within the arm of the pendulum.

As a remedy, for the example of the adaptive control of an underactuated Classical Mechanical System with singular equations of motion an ancillary tuning strategy was introduced in [19] in which only one of the three adaptive parameters of the novel MRAC controller was adaptively tuned. The tuning strategy applied was quite heuristic, and it may have countless variants.

The aim of the present paper is to analyze various tuning possibilities for the novel MRAC control for two interesting, strongly nonlinear physical paradigms as follows: a) a pendulum of uncertain mass center point that seemingly behaves as a SISO system with a not modeled coupled internal degree of freedom the actual state of which drastically

influences the behavior of the observed and controlled degree of freedom [Fig. 1]; b) the underactuated system already studied in [19] [Fig. 2].

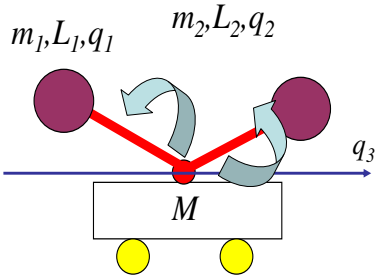

Figure 2: The structure of the underactuated system: a cart + double pendulum (q_3 does not have own drive, the motion along it can be controlled by directly controlling the motion of the counterweights m_1 and m_2 along their rotary axles q_1 and q_2).

The analysis happened by numerical computations by the use of the SCILAB-5.1.1 version and its SCICOS ver. 4.2 co-simulator packages. They can freely be used for research purposes. These software packages automatically select the appropriate numerical integration method depending on the observed stiffness of the problem under consideration. The paper is structured as follows. In Section 2 the controller and the stabilization tuning are briefly described. In Section 3 the simulation results are presented, and the paper is completed by the conclusions in Section 4.

2. The RFPT-based Adaptive Controller with Parameter Tuning

The basic idea of the novel MRAC controller is outlined in Fig. 3.

Assume that on purely kinematical basis we prescribe a trajectory tracking policy that needs a desired acceleration for certain axes as $\ddot{q}^D$. From the behavior of the reference model for that acceleration we can calculate the physical agent (force or torque) that could result in the response Q^D for the reference model. The direct application of this Q^D value for the actual system could result in different response since its physical behavior differs from that of the reference model. Therefore it can be "deformed" into a "*required*" Q^{Req} value that directly can be applied to the actual system. Via substituting the realized response of the actual system $\ddot{q}$ into the reference model the "realized control action" Q^R can be obtained instead of the "desired one" Q^D [Fig. 3]. Our aim is to find the proper deformation by the application of which Q^R well approaches Q^D while the prescribed nominal trajectory is precisely tracked, that is *at which the controlled system seems to behave as the reference system and the trajectory tracking is precise, too*. The proper deformation may be found by the application of an iteration as follows.

Consider the iteration generated by some function as $Q_{n+1}^{Req} = G(Q_n^{Req},Q_n^R,Q_{n+1}^D)$ in which n is the index of the control cycle. For slowly varying scenario Q^D can be considered to be constant. In this case the iteration is reduced to $Q_{n+1}^{Req} = G(Q_n^{Req},Q_n^R,Q^D)$ that must be made convergent to some Q_*^{Req}. One possibility for that is the application of contractive maps in the arrays of real numbers that result in Cauchy Sequences that are convergent in a complete linear normed space. By using the norm–inequality, for a convergent iterative sequence $x_n \to x_*$ it is obtained that $Q_*^{Req} = G(Q_*^{Req},Q^R(Q_*^{Req}),Q^D)$ since

$$\begin{gathered} ||G(x_*,Q^D) - x_*|| = ||G(x_*,Q^D) - x_n + x_n - x_*|| \leq \\ \leq ||G(x_*,Q^D) - x_n|| + ||x_n - x_*|| = \\ = ||G(x_*,Q^D) - G(x_{n-1},Q^D)|| + ||x_n - x_*||. \end{gathered} \tag{5}$$

It is evident from (5) that if G is continuous then the desired fixed point is found by this iteration because in the right hand side of (5) both terms converge to 0 as $x_n \to x_*$. The next question is giving the necessary or at least a *satisfactory condition* of this convergence. It also is evident that for this purpose contractivity of $G(\bullet)$, i.e. the property that $||G(a) - G(b)|| \leq K||a - b||$ with $0 \leq K < 1$ is satisfactory since it leads to a *Cauchy Sequence* ($||x_{n+L} - x_n|| \to 0 \; \forall L \in \mathbb{N}$):

$$\begin{gathered} ||x_{n+L} - x_n|| = ||G(x_{n+L-1}) - G(x_{n-1})|| \leq ... \\ \leq K^n||x_L - x_0|| \to 0 \;\; as \quad n \to \infty \end{gathered} \tag{6}$$

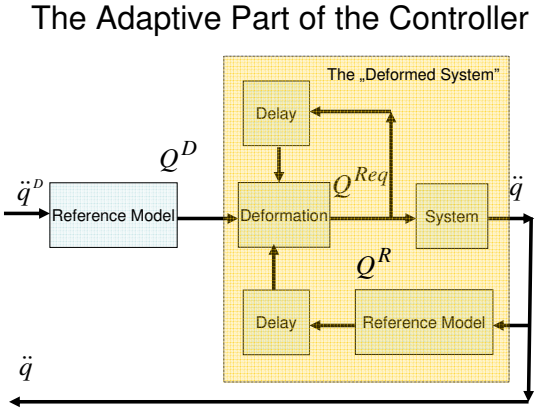

Figure 3: The block scheme of the "novel" MRAC controllers.

For the role of function $G(x,Q^D)$ a novel fixed point transformation was introduced for *Single Input – Single Output (SISO) Systems* that is rather "robust" as far as the dependence of the resulting function on the behavior of $f(\bullet)$ is concerned (7). This robustness can approximately be investigated by the use of an affine approximation of $f(x) := Q^R(x)$ in the vicinity of x_* and it is the consequence of the strong nonlinear saturation of the sigmoid function $\tanh(x)$:

$$\begin{array}{c} G(x,Q^D) := (x+K) \times \left[1 + B\tanh(A[f(x) - Q^D])\right] - K \\ \texttt{if} f(x_*) = Q^D \ \texttt{ then } \ G(x_*|Q^D) = x_*, \quad G(-K,Q^D) = -K, \\ G(x_*,Q^D)' := (\partial G/\partial x \ \texttt{ at } \ x = x_*) = (x_* + K)ABf'(x_*) + 1. \end{array} \tag{7}$$

Equation (7) evidently has a proper (x_*) and a false ($-K$) fixed point, but by properly manipulating the control parameters A, B, and K the condition $|G'(x_*,Q^D)| < 1$ can be guaranteed and the good fixed point can be located within the basin of attraction of the procedure. This means that the iteration can have considerable speed of convergence even nearby x_*, and the strongly saturated $\tanh$ function can make it robust in its vicinity, that is the properties of $f(x)$ have not too much influence on the behavior of G. [It can be noted that instead of the $\tanh$ function any sigmoidal function with the property of $\sigma(0) = 0$, e.g. $\sigma(x) := x/(1+|x|)$ can be similarly applied, too.]

The idea of keeping the iteration convergent by manipulating the width of the basin of attraction to the good fixed point can be developed in the following manner for SISO systems. On the basis of the available rough system model a simple PID controller can be simulated that reveals the order of magnitude of the occurring responses. Parameter K can be so chosen for which the $x+K$ values are considerable negative numbers. Depending on $sign(f')$ let $B = \pm 1$ and let $A > 0$ be a small number for which $|\partial G(x,Q^D)/\partial x| \approx 1 - \varepsilon_{goal}$ for a small $\varepsilon_{goal} > 0$. For Q^D varying in time the following estimation can be done in the vicinity of the fixed point when $|x_n - x_{n-1}|$ is small: $x_{n+1} - x_n = G(x_n,Q_n^D) - G(x_{n-1},Q_{n-1}^D) \approx \frac{\partial G(x_{n-1},Q_{n-1}^D)}{\partial x}(x_n - x_{n-1}) + \frac{\partial G(x_{n-1},Q_{n-1}^D)}{\partial Q^D}(Q_n^D - Q_{n-1}^D)$. Since from the analytical form of $\sigma(x)$ the term $\frac{\partial G(x_{n-1},Q_{n-1}^D)}{\partial Q^D}$ is known, and the past "desired" inputs as well as the arguments of function G are also known, this equation can be used for real–time estimation of $\frac{\partial G(x_{n-1},Q_{n-1}^D)}{\partial x}$. The quantity ε_{goal} can be tried to be fixed around -0.5 by tuning parameter A for which various possibilities are available. For the $\sigma(x) := x/(1+|x|)$ choice a "*moderate tuning strategy*" according to (8)

$$\begin{array}{l} d_n := x_n - x_{n-1}, \ \ d_{n-1} := x_{n-1} - x_{n-2} \ \ h_n := BA\sigma'\left(A(f_{n-1} - x_{n-1}^d)\right) \\ d_n^d := Q_n^D - Q_{n-1}^D \ \ \varepsilon := \left(d_n + (x_{n-1} + K)h_n d_n^d\right)/d_{n-1} - 1 \\ \texttt{if } \ \varepsilon - \varepsilon_{goal} < 0 \ \texttt{ then } \quad \dot{A} = \kappa_1\alpha_1\left(\sigma(\varepsilon - \varepsilon_{goal}) + \kappa_2 sgn(\varepsilon - \varepsilon_{goal})\right) \\ \texttt{else } \ \dot{A} = \alpha_1\left(\sigma(\varepsilon - \varepsilon_{goal}) + \kappa_2 sgn(\varepsilon - \varepsilon_{goal})\right) \end{array} \tag{8}$$

with $\kappa_1 > 1$, κ_2,$\alpha_1 > 0$ parameters [$\sigma'(x)$ refers to the derivative of the function $\sigma(x)$], and a more drastic "*exponential tuning*" according to (9)

$$\texttt{if } \varepsilon - \varepsilon_{goal} < 0 \texttt{ then } \dot{A} = -\kappa_3 \alpha_2 A \texttt{ else } \dot{A} = \alpha_2 A \tag{9}$$

with $\kappa_3 > 1$, $\alpha_2 > 0$ parameters can be prescribed. In (8) κ_1 and in (9) κ_3 corresponds to faster decrease than increase in A since it was observed that decreasing A introduces little fluctuations in the consecutive x_n values in a discrete approach, therefore it is more expedient to quickly step over this fluctuating session by fast decrease in A. In the next section simulation results will be presented for the application of these tuning strategies.

3. Simulation Results

In the simulations the maximum allowed discrete time-step of integration was set to $10^{-4}s$, while the cycle–time of the controller was assumed to be $1ms$. The "digital nature" of the controller was modeled by the use of event clocks and sample holders. The nominal trajectory was a 3rd order periodic spline function of time.

At first the control of the underactuated system in Fig. 2 is considered in which the horizontal position of the arms of the counterweights is critical since their motion does not generate horizontal force components. Therefore one of the weights were kept at vertical position while the other weight's reaction force was used for motion control. Whenever the active weight approached the critical position it was directed back to the horizontal direction and the other one's reaction force was used for active control. The here applied solution used a finer combination of the reaction forces of the counterweights than that in [19]. The *actual numerical values* were: $M = 20\,kg$, $m_1 = 8\,kg$, $m_2 = 8\,kg$, $L_1 = L_2 = 2\,m$, $g = 9.81\,m/s^2$. The reference model had the parameters as $\hat{M} = 18\,kg$, $\hat{m}_1 = 4\,kg$, $\hat{m}_2 = 6\,kg$, $\hat{L}_1 = \hat{L}_2 = 2\,m$, $\hat{g} = g$. In the forthcoming figures the following "*naming conventions*" are used: the "*nonadaptive*" version uses the approximate dynamic model with a simple kinematically designed PID controller; the "*classic*" version uses the rule defined in (8) with $\kappa_1 = 2$, $\alpha_1 = 2.5/s$, and $\kappa_2 = 0$; the "*speedy*" version uses the rule defined in (8) with $\kappa_1 = 2$, $\alpha_1 = 2.5/s$, and $\kappa_2 = 0.1$ (that also yields some finite tuning speed in the close vicinity of zero error; and the "*exponential*" version corresponds to (9) with $\alpha_2 = 2.5/s$ and $\kappa_3 = 3$.

Figure 4 describes the trajectory tracking properties of the controllers. The tracking errors of the nonadaptive solution reveal the significance of the modeling errors that are finely reduced by the adaptive solutions. The same holds for the phase trajectory tracking and acceleration tracking given in Figs. 5 and 6.

The realization of the "*MRAC principle*" is well revealed by Fig. 7 and in details by Fig. 8. In the non-adaptive case (and in the first $0.1\,s$ long phase of the adaptive ones before switching on adaptivity) $Q^D \equiv Q^{Req}$, that is the desired values are exerted to the system without any deformation. Accordingly the lines of the "exerted (required)" components as Q_1^{Req}=(longdash dot: — . — . —), Q_2^{Req}=(bigdash dot: — .. — .. —) cannot be seen in the non-adaptive phase: they are exactly covered by the "desired" Q_1^D=(solid: —), Q_2^D=(bigdash longdash: — - — -) curves. After reaching the adaptive phase the "pairs"

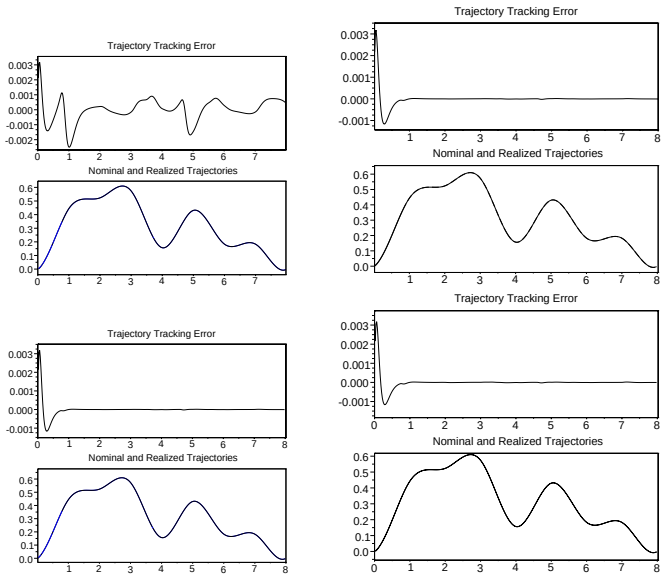

Figure 4: Trajectory tracking of the underactuated system ($q_3\,m$ vs. time in s units) of the "novel" MRAC controller with various tuning methods: "*nonadaptive*": upper left, "*classic*": upper right, "*speedy*": lower left, "*exponential*": lower right charts.

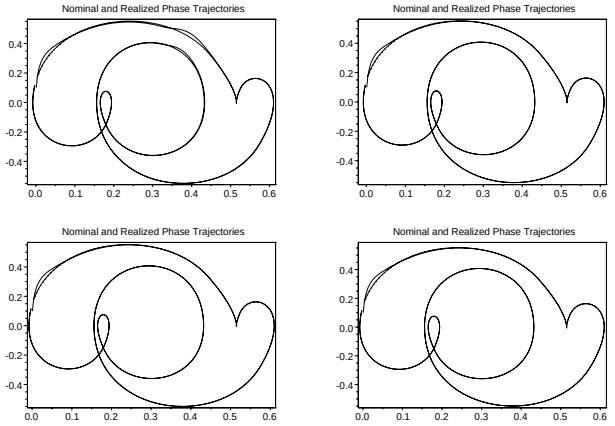

Figure 5: Phase trajectory of the underactuated system ($\dot{q}_3\,m/s$ vs. $q_3\,m$) tracking of the "novel" MRAC controller with various tuning methods: "*nonadaptive*": upper left, "*classic*": upper right, "*speedy*": lower left, "*exponential*": lower right charts.

are reorganized: the $Q^D \approx Q^R$ situation is finely approached (the "recalculated" values well approximate the "desired" ones). Accordingly the Q_1^D=(solid: —) curve drains into itself the recalculated Q_1^R=(dash: — — —) curve, as well as the Q_2^D=(bigdash longdash: — - — -) curve attracts the Q_2^R=(dash dot: - - -) line, while the seriously deformed "exerted" curves Q_1^{Req}=(longdash dot: — . — . —) and Q_2^{Req}=(bigdash dot: — .. — .. —) remain "single".

Figure 9 reveals that the parameter tuning is more or less consistent (no exact setting is necessary for A), and the names of the tuning methods well mirror their properties.

The other system under consideration, i.e. the pendulum of Fig. 1 shows different kind of problem with its dynamically coupled subsystem not known by the controller. In its control higher dynamic ranges have to be covered for which a kind of noise–filtering technique would be useful. (While nearby the fixed point (7) is not very sensitive to these noises, the tuning rules (8) and (9) that contain different past values evidently may be noise–sensitive.) In the case of a digital controller these noises can be modeled by adding random disturbance terms to the simulated/observed second time–derivatives. Any linear noise filter can be modeled by an integral or a sum in the discrete approximation as

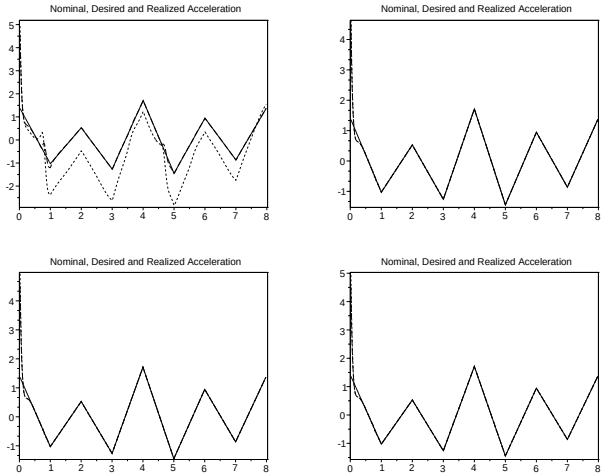

Figure 6: Acceleration tracking of the underactuated system ($\ddot{q}_3$ m/s^2 vs. time in s units) of the "novel" MRAC controller with various tuning methods: "*nonadaptive*": upper left, "*classic*": upper right, "*speedy*": lower left, "*exponential*": lower right charts; {line attributes: nominal $\ddot{q}_3^N$=(solid: —), desired $\ddot{q}_3^D$=(dash: — — —), realized $\ddot{q}_3$=(dash dot: - - -) }.

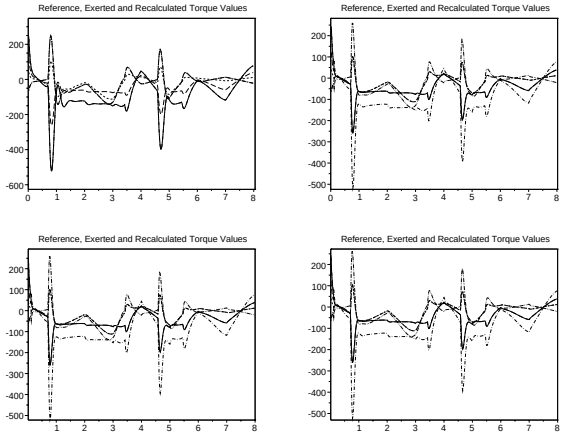

Figure 7: The torque components of the underactuated system (in Nm unit) vs. time (in s unit) of the "novel" MRAC controller with various tuning methods: "*nonadaptive*": upper left, "*classic*": upper right, "*speedy*": lower left, "*exponential*": lower right charts; {line attributes: "*desired torque components*" Q_1^D=(solid: —), Q_2^D=(bigdash longdash: — - — -); "*recalculated torque components*" Q_1^R=(dash: — — —), Q_2^R=(dash dot: - - -); "*exerted (required) torque components*" Q_1^{Req}=(longdash dot: — . — . —), Q_2^{Req}=(bigdash dot: — .. — .. —)}.

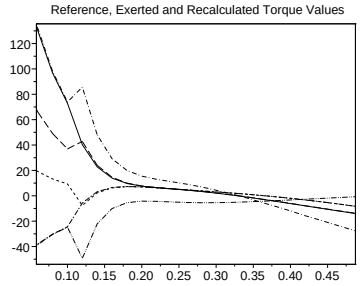

Figure 8: Fine details of the torque components of the underactuated system (in Nm unit) vs. time (in s unit) of the "novel" MRAC controller for the "*classic*" tuning method {line attributes: "*desired torque components*" Q_1^D=(solid: —), Q_2^D=(bigdash longdash: — - — -); "*recalculated torque components*" Q_1^R=(dash: — — —), Q_2^R=(dash dot: - - -); "*exerted (required) torque components*" Q_1^{Req}=(longdash dot: — . — . —), Q_2^{Req}=(bigdash dot: — .. — .. —)}.

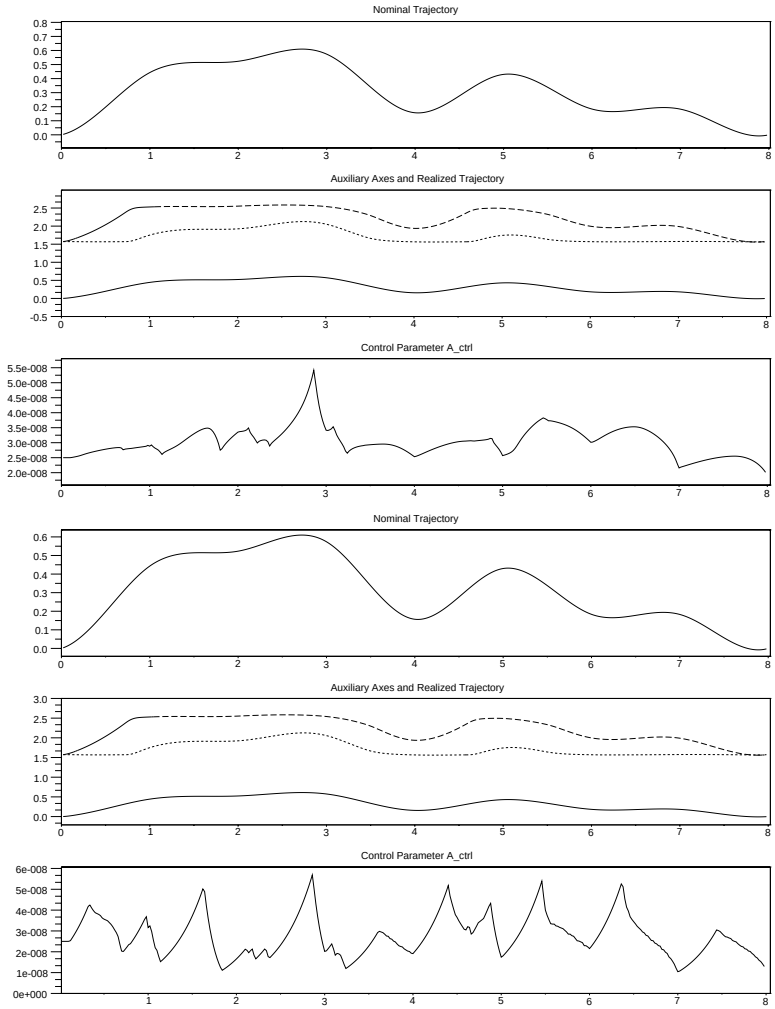

Figure 9: The motion of the axles of the underactuated system and the tuned parameter vs. time (in s unit) of the "novel" MRAC controller with two typical tuning methods: the "*classic*", and the '*exponential*'tuning {line attributes: nominal trajectory=(solid: — in the upper segments), realized trajectory=(solid: —), auxiliary trajectories: (dash: — — —) and (dash dot: - - -) in the middle segments, the control parameter: (solid: —) line in the lower segments of the graphs.

$$\tilde{f}(t) := \int_0^\infty F(\tau) f(t-\tau) d\tau \quad \text{or} \quad \tilde{f}_k := \sum_{i=0}^\infty F_i f_{k-i} \tag{10}$$

with some monotone decreasing function $F(\tau)$ or discrete weights F_k that normally converge to zero as τ,$k \to \infty$. Normally some weighted average can be calculated for a period corresponding to shorter or longer "memory". In the case of a discrete controller the smallest memory is needed for the very simple solution $F_k := \beta^k(1-\beta)$ with $0 < \beta < 1$ that can be calculated by a single buffer P according to the updating rule $P_{n+1} = \beta P_n + f_{k+1}$, $\tilde{f}_{k+1} = (1-\beta)P_{k+1}$. The actual value of β directly influences the "memory" of the system: larger value corresponds to longer memory than a smaller one. In the forthcoming simulation results the $\beta = 0.85$ value was chosen.

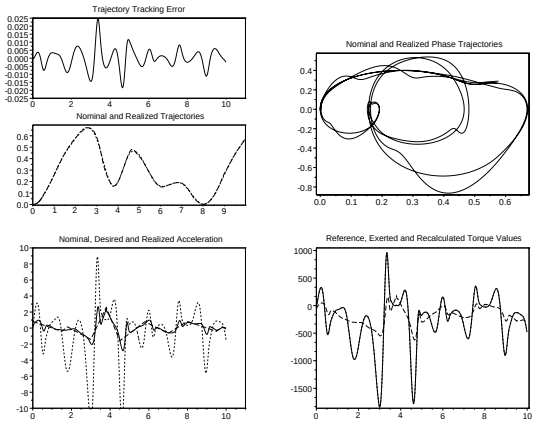

Figure 10: Some results belonging to the nonadaptive control of the pendulum {line attributes: tracking error=(solid: —), nominal motion q_1^N=(dash: — — —), realized motion q_2=(dash dot: - - -); realized acceleration $\ddot{q}_1$=(solid: —), desired acceleration $\ddot{q}_1^D$=(dash dot: - - -), nominal acceleration $\ddot{q}_1^N$=(dash: — — —) line; desired torque from the reference model Q^D=(solid: —), the deformed exerted (required) torque Q^{Req}=(dash dot: - - -) (exactly covering each other), and the torque recalculated from the reference model Q^R=(dash: — — —)}.

Figure 10 describes the operation of the simple PID controller without adaptive deformation. It can well be seen that the difference between the "exact model" and "reference model" effects significant tracking error. (According to Fig. 1 $\Theta = 30\,kg \cdot m^2$, $C = 50\,kg \cdot m^2$, $m = 50\,kg$, $k = 1000\,N/m$, $g = 9.81\,m/s^2$, $\mu = 0.1\,N/(m/s)$ –a viscous friction term not listed in the figure–, and $L_0 = 2\,m$, and $\hat{\Theta} = 50\,kg \cdot m^2$, $\hat{C} = 70\,kg \cdot m^2$, $\hat{m} = 20\,kg$, $\hat{k} = 100\,N/m$, $\hat{g} = 10\,m/s^2$, $\hat{\mu} = 0.01\,N/(m/s)$ and $\hat{L}_0 = L_0 = 2\,m$.)

Figure 11 corresponds to the tuning defined in (8) with $\kappa_1 = 2.5$, $\alpha_1 = 10/s$, and $\kappa_2 = 1$. Its drastic improvement in the trajectory tracking and the realization of the MRAC idea is evident.

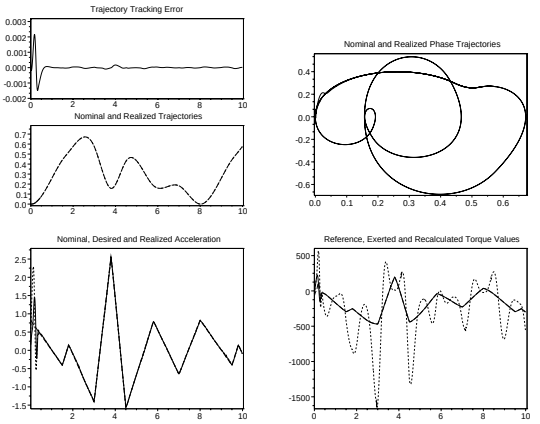

Figure 11: Some results belonging to the adaptive control of the pendulum with *"moderate tuning"* {line attributes: tracking error=(solid: —), nominal motion q_1^N=(dash: — — —), realized motion q_2=(dash dot: - - -); realized acceleration $\ddot{q}_1$=(solid: —), desired acceleration $\ddot{q}_1^D$=(dash dot: - - -), nominal acceleration $\ddot{q}_1^N$=(dash: — — —) line; desired torque from the reference model Q^D=(solid: —), the deformed exerted (required) torque Q^{Req}=(dash dot: - - -) (now quite different lines), and the torque recalculated from the reference model Q^R=(dash: — — —) (now almost exactly covering the line of Q^D)}.

Figure 12 belongs to the tuning method described in (9) with $\alpha_1 = 10/s$ and $\kappa_3 = 2$. To same holds for this result as to that described in Fig. 11.

Figure 13 reveals the variation of the tuned parameter in the case of the moderate and fast tuning for which we got comparable results again. It is important to note that the tuned parameter A varies in a great dynamic range.

It is interesting to investigate what happens in the case of the pendulum if A is set to be constant. The simulation results in Fig. 14 reveal that the active tuning really can stabilize the controller. It can be seen that at several regions strong fluctuations occur that cause considerable tracking errors. An efficient tuning can solve the problem that can be caused by an improperly chosen initial value for A. Since the proposed control method deals with observing the system's response to the excitation by the controller that depends on the properties of the nominal trajectories and external disturbances this tuning ability is a significant value.

4. Concluding Remarks

In this paper the operating principle of a novel version of the Model Reference Adaptive Controllers was briefly outlined in formal comparison with two typical representatives

of the "traditional adaptive approaches". The advantage of the new approach is that for design purposes it does not require the use of Lyapunov's ingenious "direct method" that normally needs very good mathematical skills and invention on the part of the designer. In contrast to the numerous arbitrary control parameters of the traditional approaches it applies only three parameters that can easily be set via simple simulations. The deficiency of the method is that normally it cannot guarantee global stability. It works with a local basin of attraction that can be left by the controlled system's response. To relax/amend this deficiency a simple tuning procedure was suggested for continuously modifying only one of the three adaptive parameters of this controller to keep its operation nearby the center of the basin of its convergence. For such a tuning various open possibilities are available.

In the present paper various possible tuning methods were investigated on two interesting dynamical systems. One of them was an underactuated cart and double pendulum system, the other one contained a "hidden" internal degree of freedom dynamically coupled to that under direct observation and control.

The results of the above investigations reinforced the expectation that the novel variant of the Model Reference Adaptive Controllers designed by the use of Robust Fixed Point Transformations could successfully be stabilized by tuning only one of the three control

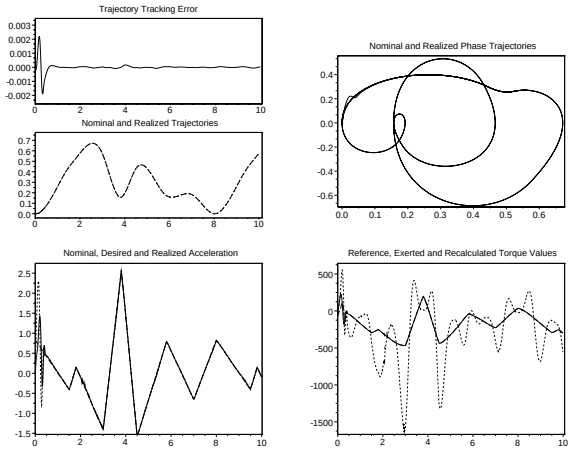

Figure 12: Some results belonging to the adaptive control of the pendulum with *"fast tuning"* {line attributes: tracking error=(solid: —), nominal motion q_1^N=(solid: —), realized motion q_2=(dash dot: - - -); realized acceleration $\ddot{q}_1$=(solid: —), desired acceleration $\ddot{q}_1^D$=(dash dot: - - -), nominal acceleration $\ddot{q}_1^N$=(dash: — — —); desired torque from the reference model Q^D=(solid: —), the deformed exerted (required) torque Q^{Req}=(dash dot: - - -), and the torque recalculated from the reference model Q^R=(dash: — — —) (almost exactly covering the line of Q^D)}.

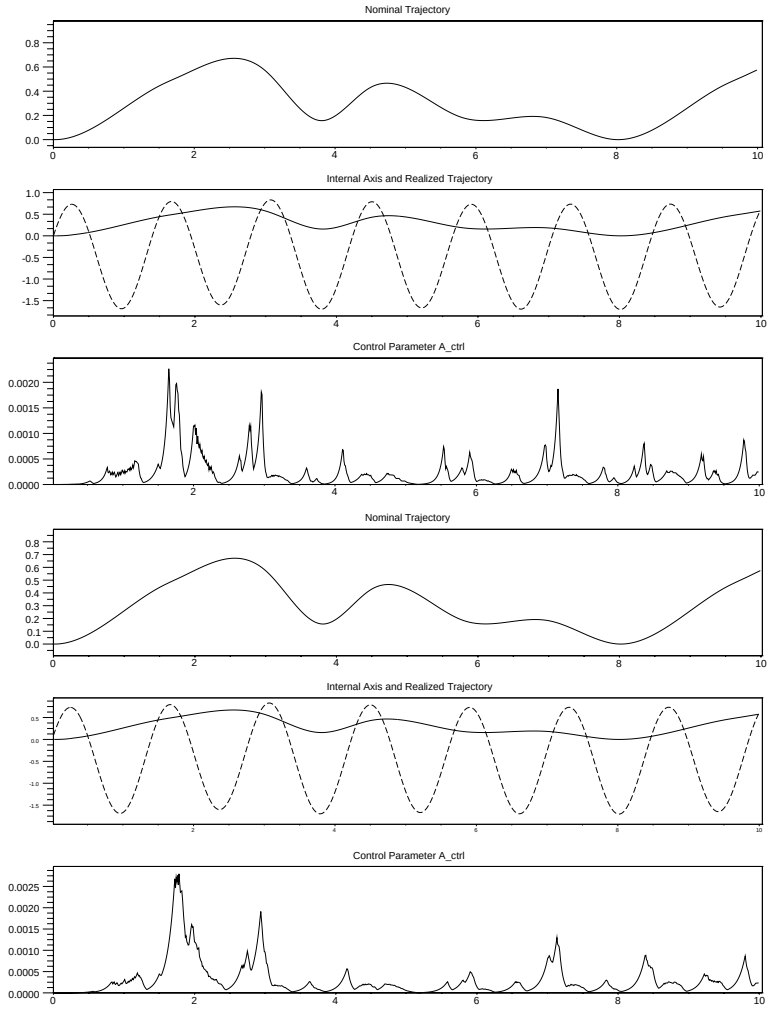

Figure 13: Some results belonging to the adaptive control of the pendulum with *"moderate tuning"* and *"fast tuning"* {line attributes: nominal trajectory: q_1^N=(solid: —), realized trajectory: q_1=(solid: —), motion of the coupled 2^{nd} axis q_2=(dash: — — —), the control parameter A=(solid: —) lines}.

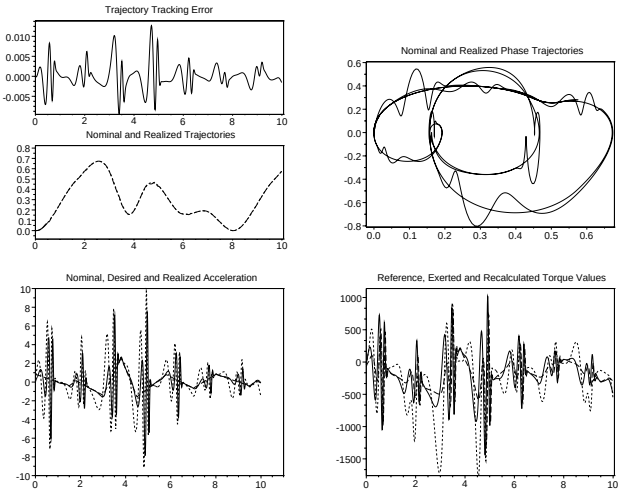

Figure 14: Some results belonging to the adaptive control of the pendulum without tuning ($A = 10^{-6} = const.$) {line attributes: tracking error=(solid: —), nominal motion q_1^N=(dash: — — —), realized motion q_2=(dash dot: - - -); realized acceleration $\ddot{q}_1$=(solid: —), desired acceleration $\ddot{q}_1^D$=(dash dot: - - -), nominal acceleration $\ddot{q}_1^N$=(dash: — — —) line; desired torque from the reference model Q^D=(solid: —), the deformed exerted (required) torque Q^{Req}=(dash dot: - - -), and the torque recalculated from the reference model Q^R=(dash: — — —)}.

parameters. In certain cases no any tuning is needed, in other cases fast tuning is required for stabilization.

In the next step of the research modeling and investigation of various noise components would be expedient that may concern the operation of the parameter tuning applied.

Acknowledgement

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