

Train Obstacle Detection System Stabilization and Stochastic Vibrations Analysis Using the Moment Lyapunov Exponent Method

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Abstract: This paper analyzes stochastic vibrations of a specialised onboard railway obstacle detection system (ODS). The observed system consists of several vision-based sensors mounted in a special housing attached to the locomotive front profile via rubber metal springs and a mounting plate. In this study, the experimental measurements of acceleration were performed in the vertical, longitudinal, and lateral direction for two positions, on the mounting plate rigidly connected to the vehicle body and inside the sensor housing. The ODS stabilization is presented with the results obtained by the moment Lyapunov exponent (MLE) method. Analytical and numerical determination of MLE is firstly presented on a simply supported Euler-Bernoulli beam. Further, the stochastic vibration analysis was performed using the experimentally obtained data. According to these values, the appropriate system parameters essential for the application of the Lyapunov theory to stochastic stability problems were firstly numerically calculated. By means of the Monte Carlo simulation method, whose example was previously shown on a simple beam, the bounds of the almost sure stability of the observed system are given according to the measured accelerations in all of the observed directions.

Keywords: Vibrations; Obstacle Detection; stability; Moment Lyapunov exponent; Monte Carlo simulation

1 Introduction

Railway digitalization and automation has become increasingly important in order to improve its competitiveness and efficiency. For autonomous train movement in GoA3/4 mode of operation it is necessary to detect obstacles in front of the moving train which could lead to collisions. Collisions with obstacles adversely affect train safety, and in most cases trains kill or severely injure any live object they collide with [1]. An obstacle detection system (ODS) is thus a necessary prerequisite for autonomous trains due to high safety requirements of the railway system.

Within the H2020 project SMART [2], an onboard ODS was developed and evaluated. The system combines different vision-based sensors integrated into a housing mounted onto the locomotive front profile. Visual sensors are able to provide very dense and detailed information about the environment. However, in practice their performance depends not just on the ambient lighting and visibility conditions [1], but also on vibrations which are transmitted from the vehicle to the sensors. For instance, edge detector performance in image processing is adversely affected by image distortion induced by vibrations [3]. The image distortion is characterized by the vehicle vibration frequency; the higher the frequency of the vehicle, the more distorted the images [4]. Image stabilization is especially important in autonomous vehicles as the better image acquisition process will increase the feasibility and reliability of the process and the subsequent analysis [5]. In order to prevent the deterioration of image quality due to vibrations, a whole new scientific and engineering area was created, which deals with image stabilization techniques. Image stabilization techniques can be classified into four major categories: optical, digital, electro-mechanical and mechanical stabilization [6]. For mechanical stabilization, the stochastic stability analysis becomes very important to ensure the almost sure stability of the system.

Rail dampers are studied in detail in the works of Kuchak *et al.* [7, 8]. The investigation of the efficiency and influential parameters of a rail damper design based on a lab-scaled model of the rail-damper system and an accurate FE model is presented in [7]. In the author's previous work [8], with the aim to improve the damper model, cone pressure theory is applied in the FE model and the sensitivity analysis is then applied to gradually improve the FE model. Paper [9] deals with the experimental investigation of the interlocking effect of crushed stone ballast material, assessing it as the relationship with the residual and dynamic stresses under the ballast layer. The study of this effect could be potentially useful for many practical problems of railway track design as well as for track maintenance issues.

The modern theory for stochastic stability analysis is based on the Lyapunov exponent (LE) and MLE determination. By using different Lyapunov theories in our previous studies [10, 11], stochastic stability and instability of elastical and viscoelastical systems is analysed for different system parameters. The work of

Arnold et al. [12] presents a small noise expansion of moment Lyapunov exponents for two-dimensional systems. Analytical and numerical determination of LE and MLE was studied in detail for various systems. In [13] using the stochastic averaging method, LE is determined for two elastically connected viscoelastic beams. The regular perturbation method for MLE determination is used in [14] where the LE and MLE of two degrees-of-freedom linear systems subjected to white noise parametric excitation are investigated. The same method is used in [15] where the dynamical behavior of two viscoelastically connected nanobeams under white noise process is analyzed. In the works of Xie [16, 17] MLE is determined for a two-dimensional system under real and bounded noise wideband stochastic processes.

One of the first studies of numerical determination of the moment Lyapunov exponents of stochastic systems using a Monte Carlo simulation approach is presented by Xie in [18]. This method is widely used, and it finds its application in almost every field of study especially when it is very hard or almost impossible to get an analytical expression for MLE. Thus, this method is presented in detail in [19], where stochastic stability of a multi-nanobeam system is analyzed for different types of axial compression on the system ends. In this study, an example of numerical determination of MLE is presented on one simple nanobeam, where numerical results are compared with the analytical ones, with the presented numerical method further using the stability analysis of a higher number of viscoelastically connected nanobeams.

This paper is organized as follows. According to the performed experiment, the essential train acceleration measurements in the vertical, lateral and longitudinal direction are given in Section 2. Section 3 presents the procedure for the analytical and numerical determination of MLE. Using the Euler-Bernoulli beam theory this example is given on an axially compressed simply supported elastic beam. After the verification of the numerical method for MLE determination, this method is further applied for ODS stochastic vibrations analysis (Section 4), where the influence of the viscoelastic medium on the system stabilization is presented. The paper ends with the conclusion section containing the final remarks.

2 Experimental Determination of ODS Vibrations

Within the H2020 project SMART [2], a prototype of the onboard ODS for freight trains was developed. An integrated vision sensor-based onboard ODS for freight trains combines multiple vision sensors integrated into the ODS housing mounted on the locomotive front profile as shown in Figure 1. The ODS housing is mounted on the locomotive front profile to the mounting plate rigidly connected to the locomotive body. The ODS housing is connected to the mounting plate via 4 rubber-metal bushings (Trelleborg M50-40), which passively suppress the vibrations

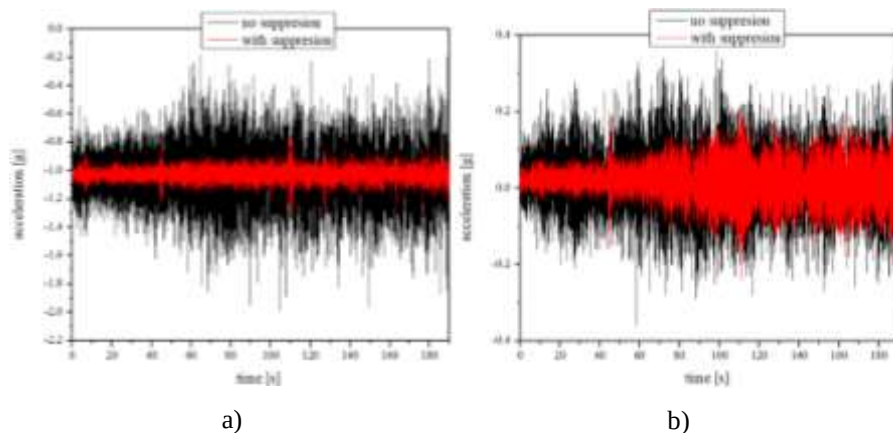
transmitted from the locomotive body to the ODS housing. The design of the housing and the passive vibration suppression system is described in detail in reference [20].

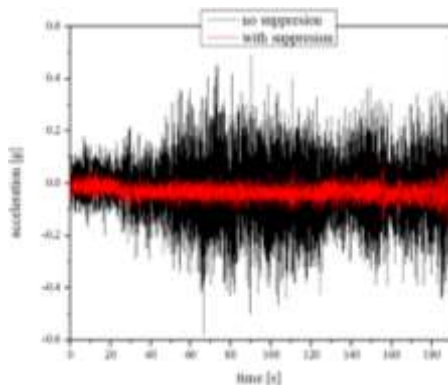


Figure 1

The ODS sensor housing mounted onto the locomotive front profile [1]

The performance of the vibration suppression system was measured during the operational train run at the speed of 80 km/h on a straight portion of the track. The accelerations were measured at two positions - on the mounting plate near the connection point with the rubber metal bushing and in the ODS housing directly above the connection point of the ODS housing with the rubber metal bushing with two triaxial IMUs. The experimental results are shown in Figure 2.





c)

Figure 2

Experimentally determined performance of vibration suppression in three directions: a) vertical, b) lateral, c) longitudinal

3 MLE of a Simply Supported Euler-Bernoulli Beam

For stochastic vibration analysis of the system described in the previous section the MLE numerical determination is suggested. Basic definitions of stochastic stability are the simple or almost sure stability and the stability in the mean of the p -th order, which is based on the concept of the Lyapunov exponent given in Arnold et al. [12]. The almost sure stability is described by the maximal Lyapunov exponent defined as:

$$\lambda_q = \lim_{t \rightarrow \infty} \frac{1}{t} \ln \|q(t; q_0)\| \quad (1)$$

where $q(t; q_0)$ is the solution process of a linear dynamic system. It gives the exponential growth rate of the solution. If $\lambda_q < 0$, then, by definition, $\|q(t; q_0)\|^p \rightarrow 0$ as $t \rightarrow \infty$, the solution is almost surely stable, and $\lambda_q > 0$ implies the instability of the solution in the almost sure sense. The exponential growth rate $E\|q(t; q_0)\|^p$, where E denotes the expectation, is provided by the moment Lyapunov exponent defined as

$$\Lambda_q(p, q_0) = \lim_{t \rightarrow \infty} \frac{1}{t} \ln E \|q(t; q_0)\|^p \quad (2)$$

If $\Lambda_q(p, q_0) < 0$, then by definition, $E\|q(t; q_0)\|^p \rightarrow 0$ as $t \rightarrow \infty$, and those conditions are referred to as the p -th moment stability. MLE provides us with finer stability properties of the random dynamic system.

The example of MLE method is given for the simply supported Euler-Bernoulli beam where the typical beam element is given in Figure 3.

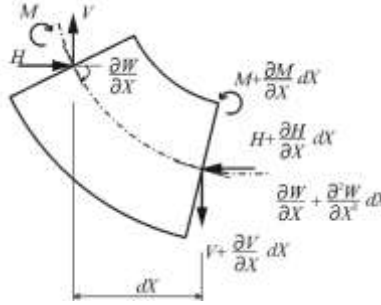


Figure 3

Typical beam element

Following the Euler-Bernoulli method, which is based on a moving field:

$$U = U(X, T) = -Z \frac{\partial W(X, T)}{\partial X}, \quad W = W(X, T) \quad (3)$$

the only deformation different from zero is the dilatation in the X direction:

$$\varepsilon_x = \varepsilon = \frac{\partial U}{\partial X} = -Z \frac{\partial^2 W}{\partial X^2}, \quad (4)$$

Now, according to the beam element from Figure 3 the following dynamical equations can be written:

$$\begin{aligned} m\ddot{a} &= \sum_{i=1}^n \vec{F}_i \Rightarrow \rho A \frac{\partial^2 W}{\partial T^2} + c_1 \frac{\partial W}{\partial T} = \frac{\partial W}{\partial X} - q_1 + q_2, \\ J\ddot{\phi} &= \sum_{i=1}^n \vec{M}_i \Rightarrow \frac{\partial M}{\partial X} - V - H \frac{\partial W}{\partial X} = 0, \end{aligned} \quad (5)$$

where ρ is the mass density, c_1 is the viscous damping coefficient, q_1 and q_2 are the continual loads on the lower and upper side of the beam, and V and H are the components of the main vector of internal forces.

The acting moment from equation (5) is:

$$M = -EI \frac{\partial^2 W}{\partial X^2} \quad (6)$$

A combination of equations (3), (4) and (5) gives:

$$\rho A \frac{\partial^2 W}{\partial T^2} + c \frac{\partial W}{\partial T} + \frac{\partial}{\partial X} \left(\frac{\partial H}{\partial X} \right) + q_1 - q_2 + \frac{\partial^2}{\partial X^2} \left(EI \frac{\partial^2 W}{\partial X^2} \right) = 0, \quad (7)$$

For a time-dependent axially loaded beam with the constant cross-section, $H=F(T)$, the relation (7) becomes:

$$\rho A \frac{\partial^2 W}{\partial T^2} + c \frac{\partial W}{\partial T} + F(T) \frac{\partial^2 W}{\partial X^2} + EI \frac{\partial^4 W}{\partial X^4} = 0, \quad (8)$$

where T is the time, EI presents the beam bending stiffness and $F(T)$ is the time-dependent stochastic process.

Finally, by substituting the following parameters:

$$T = k_t t, \quad W = Lw, \quad X = Lx, \quad k_t = L^2 \sqrt{\frac{\rho A}{EI}},$$

$$2\varepsilon\beta = \frac{ck_t}{\rho A}, \quad \sqrt{\varepsilon} f(t) = \frac{F(t)}{\rho AL^2},$$

in equation (8) the following nondimensional form of the beam is obtained:

$$\frac{\partial^2 w}{\partial t^2} + 2\varepsilon\beta \frac{\partial w}{\partial t} + \sqrt{\varepsilon} f(t) \frac{\partial^2 w}{\partial x^2} + \frac{\partial^4 w}{\partial x^4} = 0, \quad (9)$$

Now, by substituting the solution

$$w = q \sin(\pi x), \quad (10)$$

in (9), the discretized form of the equations of motions of the beam system in the first mode becomes:

$$\ddot{q} + 2\varepsilon\beta \dot{q} + \pi^4 q - \sqrt{\varepsilon} \pi^2 f_1(T) q = 0. \quad (11)$$

In the previous equation, β and ε are the damping coefficient and the small fluctuation parameter, respectively.

Finally, according to [19], the analytical expression for MLE for system (11) is obtained in the following form:

$$\Lambda(p) = \varepsilon \left[\frac{p(3p+10)}{64} \sigma^2 - p\beta \right]. \quad (12)$$

The parameter σ^2 in equation (12) is the intensity of the white noise process.

Now, according to the procedure in our previous papers, MLE is numerically calculated with the aim of comparing it with the results from equation (12).

As presented in reference [19], the system given with equation (11) is simulated 2000 times where the states q and $\dot{q}$ are estimated after each simulation and replaced in equation (2) to obtain a numerical value of MLE. The simulation lasts for 10 s, with the step size being 0.1 s. The numerically obtained results are now used to compare with the results from (12) and presented in Figure 4 with the aim of approving the suggested numerical method for further use.

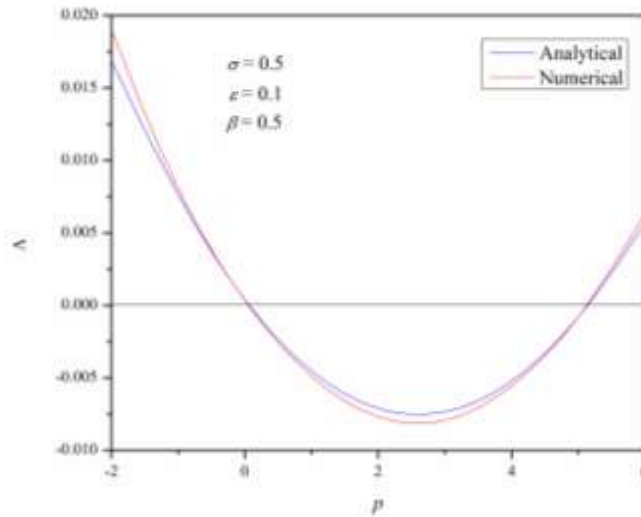


Figure 4

Comparison of analytical and numerical results of MLE for eq. (12)

3 Numerical Results and Discussion

A Monte Carlo simulation numerical method for determining the p -th MLE of stochastic systems is now applied to the obtained experimental results. This method is very useful in the cases when it is very hard or impossible to obtain an analytical expression for MLE.

According to the performed experiment, the averaged measured accelerations and their standard deviations in the vertical, longitudinal, and lateral direction are used to obtain the parameters important for the numerical determination of MLE given by expression (2). According to the Monte Carlo simulation method and using the MATLAB software these parameters were determined for 2000 calculated and integrated values in the range of the standard acceleration deviations, where 2000 different norms in equation (2) were calculated with the aim of numerically obtaining MLE for the analyzed system.

Firstly, stability analysis is given for stochastic vibrations in the vertical direction. This analysis is the most important in the study of the system stability because of the largest system vibrations which occur in this direction. According to the available experiment dataset, the results in this direction are firstly separated in seven points as the train velocity grows from 20 km/h (point-I) to 80 km/h (point-VII) and presented in Figure 5. This figure presents the averaged accelerations and their standard deviations calculated for both observed cases (with and without suppression).

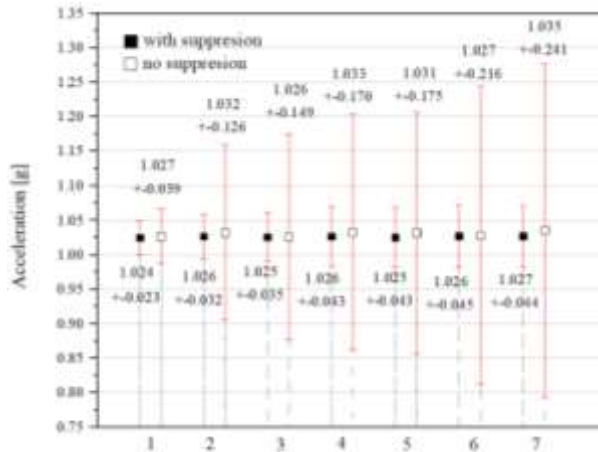


Figure 5

Averaged acceleration values and their standard deviations in the observed period

Now, by means of the Monte Carlo simulation method, the data from this figure are firstly integrated in the ranges in their standard deviations to determine the input states essential for the numerical determination of MLE.

By following the procedure for MLE numerical determination in the previous section and according to the experimental measurement in the vertical direction, the following numerical values of MLE are obtained and presented in Figure 6 and Figure 7. These figures present the variation in the almost sure stability regions for the observed train acceleration period. The results are presented in the 3D plane of MLE λ , norm degree p and train change in velocity (20-80 km/h).

Figure 6 shows the stability surface according to the vibrations of the train. For every tested point the results show significantly larger values of MLE even for low values of the norm degree p .

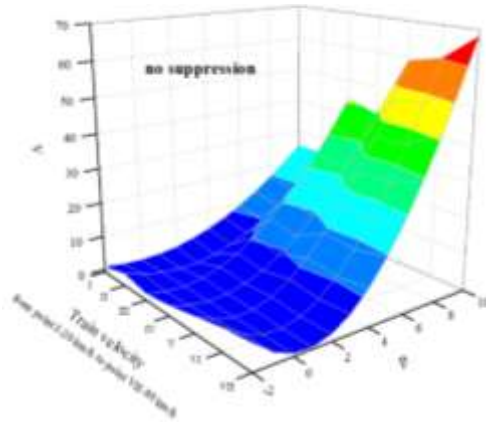


Figure 6

Stability surface according to the measurement values in the vertical direction (without suppression)

The results in Figure 7 are given for the case where suppression is applied. Contrary to Figure 6, significantly lower values of MLE are evident. Also, parameter Λ is negative in the range from $p=0$ to $p=7$, which guarantees the system stability. In comparison to Figure 6, the stabilization is especially evident for the largest values of the norm degree p , which are presented with very big differences in Λ ($p=9,10$).

The vibrations in the lateral and longitudinal directions are much smaller as evident from the experiment results. Thus, the system stabilization in these directions is observed only for the highest train velocity (corresponding values of point 7 from Figure 6 in the lateral and longitudinal direction).

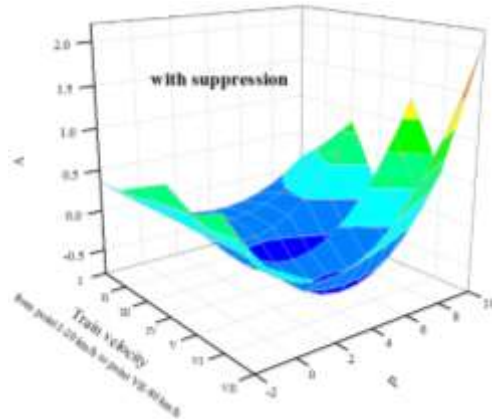


Figure 7

Stability surface according to the measurement values in the vertical direction (with suppression)

Figure 8 presents the boundaries of the almost sure stability where the red line presents the lateral stability of the train and the blue line presents the system stabilization after suppression is applied. As in Figures 6 and 7, the influence of viscoelastic suppression is also evident in this figure, where it can be seen that MLE growth for the highest values of the norm degree is almost negligible.

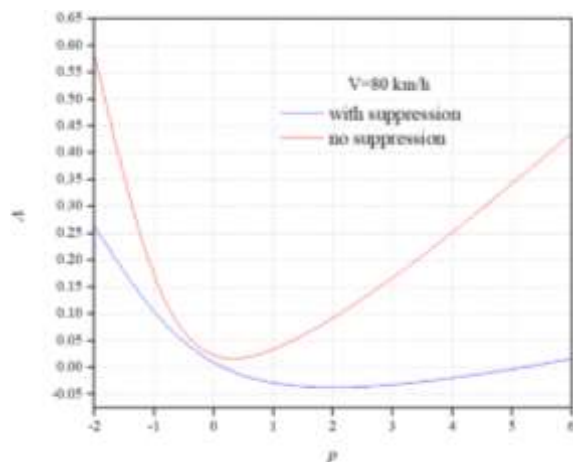


Figure 8
Stability boundaries in the lateral direction

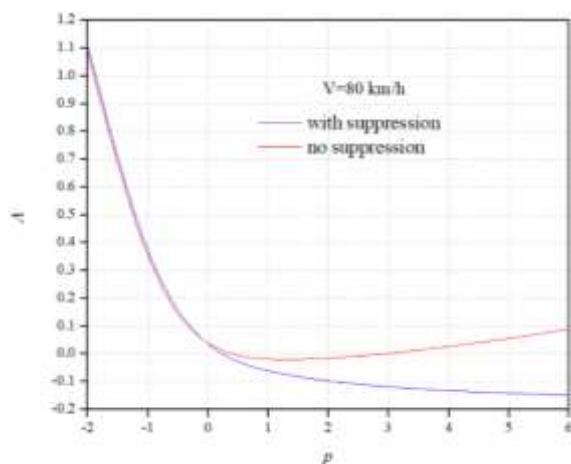


Figure 9
Stability boundaries in the longitudinal direction

Stability boundaries in the longitudinal direction are presented in Figure 9. This direction yields the smallest vibrations. When suppression is applied (blue line), the values of MLE are negative for all positive values of degree p .

Conclusions

This paper deals with stochastic vibrations of a railway ODS. The camera vibrations play a very important role because of their impact on the final picture quality. The experimental study considered the measuring of accelerations at two positions to assess the performance of the applied vibration suppression. In order to present the influence of the viscoelastic medium on ODS stabilization, the MLE method was applied. This method was firstly presented and verified on a simple Euler-Bernoulli beam with the aim of applying it to the analysis of the observed stochastic system. According to the experimental data and by means of the MLE method, the regions of the almost sure stability were given in the transversal, longitudinal and lateral directions. For this purpose the Monte Carlo simulation method was applied. This numerical method was found to be very useful in the stability problem analysis when it is hard or impossible to obtain an analytical solution, which was the case here. According to the performed numerical study, it is evident that the suggested viscoelastic element for the optical system successfully deals with stochastic vibrations where the resulting stabilization of the optical system is presented in all observed directions. This is most prominent in the study of the vertical direction dynamics, where the largest disturbances occur.

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References

- [1] Ristić-Durrant, D., Haseeb, M. A., Banić, M., Stamenković, D., Simonović, M. and Nikolić, D., (2021) SMART On-Board Multi-Sensor Obstacle Detection System for Improvement of Rail Transport Safety. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, p. 09544097211032738, DOI: 10.1177/09544097211032738
- [2] <https://cordis.europa.eu/project/id/730836>, accessed on 26.09.2021
- [3] Pavlović, M., Nikolić, V., Simonović, M., Mitrović, V. and Ćirić, I., (2019) Edge Detection Parameter Optimization Based on the Genetic Algorithm for Rail Track Detection. Facta Universitatis, Series: Mechanical Engineering, 17(3), pp. 333-344, DOI: 10.22190/FUME190426038P
- [4] Abolmaali, A., Fernandez, R., Kamangar, F., Ramirez, G. and Le, T., (2008) Vibration Reduction and Control for Traffic Cameras: Technical Report (No. FHWA/TX-08/0-5251-2)

- [5] Zhao, G. W. and Yuta, S. I., (1993) January. Obstacle Detection by Vision System for an Autonomous Vehicle. In 1993 Intelligent Vehicles Symposium, IV 1993, pp. 31-36, ISBN: 0780313704
- [6] SMART project, Deliverable 1.1. Obstacle Detection System Requirements and Specification. <https://cordis.europa.eu/project/id/730836/results>
- [7] Kuchak, A. J. T., Marinkovic, D. and Zehn, M., (2021) Parametric Investigation of a Rail Damper Design Based on a Lab-Scaled Model. *Journal of Vibration Engineering and Technologies*, 9(1), pp. 51-60, DOI: 10.1007/s42417-020-00209-2
- [8] Kuchak, A. J. T., Marinkovic, D. and Zehn, M. Finite Element Model Updating - Case Study of a Rail Damper (2020) *Structural Engineering and Mechanics*, 73(1), pp. 27-35, DOI: 10.12989/sem.2020.73.1.027
- [9] Sysyn, M., Nabochenko, O., Kovalchuk, V., Przybyłowicz, M. and Fischer, S. (2021) Investigation of Interlocking Effect of Crushed Stone Ballast During Dynamic Loading. *Reports in Mechanical Engineering*, 2(1), 65-76, DOI: 10.31181/rme200102065s
- [10] Pavlović, I., Pavlović, R. and Janevski, G., (2016) Dynamic Instability of Coupled Nanobeam Systems. *Meccanica*, 51(5), pp. 1167-1180, DOI: 10.1007/s11012-015-0278-x
- [11] Pavlović, I., Pavlović, R. and Janevski, G., (2019) Dynamic Stability and Instability of Nanobeams Based on the Higher-Order Nonlocal Strain Gradient Theory. *The Quarterly Journal of Mechanics and Applied Mathematics*, 72(2), pp. 157-178, DOI: 10.1093/qjmam/hby024
- [12] Arnold, L., Doyle, M. M. and Sri Namachchivaya, N., (1997) Small Noise Expansion of Moment Lyapunov Exponents for Two-Dimensional Systems. *Dynamics and Stability of Systems*, 12(3), pp. 187-211, DOI: 10.1080/02681119708806244
- [13] Pavlović, I., Pavlović, R., Kozić, P. and Janevski, G., (2013) Almost Sure Stochastic Stability of a Viscoelastic Double-Beam System. *Archive of Applied Mechanics*, 83(11), pp. 1591-1605, DOI: 10.1007/s00419-013-0767-0
- [14] Janevski, G., Kozić, P., Pavlović, R. and Posavljak, S., (2021) Moment Lyapunov Exponents and Stochastic Stability of a Thin-Walled Beam Subjected to Axial Loads and End Moments. *Facta Universitatis, Series: Mechanical Engineering*, 19(2), pp. 209-228, DOI: 10.22190/FUME191127014J
- [15] Pavlović, I., Pavlović, R., Janevski, G., Despenić, N. and Pajković, V., (2020) Dynamic behavior of two elastically connected nanobeams under a white noise process. *Facta Universitatis, Series: Mechanical Engineering*, 18(2), pp. 219-227, DOI: 10.22190/FUME190415008P

- [16] Xie, W. C., (2003) Moment Lyapunov Exponents of a Two-Dimensional System Under Bounded Noise Parametric Excitation. *Journal of Sound and Vibration*, 263(3), pp. 593-616, DOI: 10.1016/S0022-460X(02)01068-4
- [17] Xie, W. C., (2001) Moment Lyapunov Exponents of a Two-Dimensional System Under Real-Noise Excitation. *Journal of Sound and Vibration*, 239(1), pp. 139-155, DOI: 10.1006/jsvi.2000.3211
- [18] Xie, W. C., (2005) Monte Carlo Simulation of Moment Lyapunov Exponents. *J. Appl. Mech.*, 72(2), pp. 269-275, DOI: 10.1115/1.1839592
- [19] Pavlović, I., Karličić, D., Pavlović, R., Janevski, G. and Ćirić, I., (2016) Stochastic Stability of Multi-Nanobeam Systems. *International Journal of Engineering Science*, 109, pp. 88-105, DOI: 10.1016/j.ijengsci.2016.09.006
- [20] Banić, M., Stamenković, D., Miltenović, A., Simonović, M. and Milošević, M., (2019) Design of Housing and Vibration Suppression for Obstacle Detection System in Railways. *Proceedings of 24th International conference "Current Problems in Rail Vehicles" - PRORAIL 2019*, 1, Žilina, Slovakia, 17-19, September, pp. 23-31, ISBN 978-80-89276-58-5