We have attempted to distinguish clearly between complex resistivity phenomena and complex dielectric behavior, based on the concept of conduction current and displacement current. As a result it is possible to use ridge regression inversion to solve simultaneously for parameters describing the contribution of each phenomenon to observed spectra. In a similar manner it is possible to carry out limited identification of other important processes such as membrane polarization, double layer capacitance, and inductive electromagnetic coupling.

Perhaps the most important single use of inversion in rock properties analysis, however, is to reduce spectral curves to a few numbers, so that changes in spectra with important physical variables such as temperature, mineral composition, concentration and grain size, may be characterized and described accurately.

An outgrowth of our examination of simple relaxation models in all three domains (frequency, time and distribution function), is the capability to transform broadband measurements made in one domain to any other domain or to another type of measurement made in the same domain: for example, amplitude to phase. This is accomplished by using ridge regression inversion to fit a sum of simple relaxation models to the observed data. The determined parameters, and the known analytical form of the models in all other domains can then be used to construct any desired representation of the observed data.

d-complex resistivity, dielectric behavior, ridge regression, membrane polarization, double layer capacitance

7. Ridge regression inversion

We have spent the first part of this paper considering various proposed models for relaxation. What we will now discuss, is an inversion method which can be used to fit automatically the various proposed relaxation models to observed data. In addition to fitting the data, the inversion program provides 1) goodness-of-fit information (which can be used to help select the most appropriate model), 2) quantitative estimates of model parameters (so that the essential features of the relaxation may be plotted against other physical variables such as temperature, composition, grain size and concentration) and 3) information on the importance of each data point regarding the resolution of the total model or of individual parameters (necessary for efficient data collection and experiment design).

1 Phoenix Geophysics Limited, Willowdale, Ontario, Canada
2 Department of Geology and Geophysics, University of Utah, Salt Lake City, Utah, USA
3 U.S. Geological Survey, Denver, Colorado, USA
Manuscript received: 10 November, 1983
The algorithm

The algorithm which we have used in order to obtain estimates of model parameters, \( p \), from observed complex resistivity or dielectric data, \( y \), is

\[
\Delta p = (A^T A + \lambda I)^{-1} A^T A y
\]

where \( A \) is the matrix containing derivatives of every data point with respect to every parameter,

\[
A_{ij} = \left. \frac{\partial y_i}{\partial p_j} \right|_{p_j}
\]

\( A^T \) is the transpose of \( A \), \( I \) is the identity matrix, and \( \lambda \) is a small positive constant which gives stability to the inversion.

When \( \lambda \) is exactly equal to zero, (88) is the Newton-Raphson algorithm which is very fast, but which may diverge if \( A^T A \) is nearly singular. Thus in order to facilitate convergence in spite of a poor initial guess or poorly-determined parameters, we slightly modify \( A^T A \) before inversion [Levenberg 1944, Foster 1961, Marquardt 1963]. Since \( A^T A \) is symmetric and non-negative definite, the addition of \( \lambda \) to its diagonal elements ensures that none of the eigenvalues of the modified matrix will be zero, and therefore the matrix will not be singular. In the limit, as \( \lambda \) becomes very large, the algorithm approaches the gradient method, which always converges but which is very slow. In order to maintain stable yet fast convergence, \( \lambda \) must usually be changed at each new iteration during the inversion. Computer algorithms which accomplish this are known as ridge regression methods.

The general inversion program used in this research on rock properties evolved from earlier work by Smith [1975] and was later modified to incorporate ridge regression and simultaneous inversion of multiple data sets [Pelton et al. 1974, Pelton et al. 1976]. Since formulation is in terms of discrete parameters, the inversion program is ideally suited to problems which are naturally posed in terms of discrete parameters (such as simple relaxation models); however reasonable success has also been achieved in using the program to determine continuous functionals such as resistivity versus depth [Rijo et al. 1977, Petrick et al. 1977]. The main factor influencing effective use of the inversion technique is appropriate parameterization of the problem.

Parameterization

Perhaps the most fundamental consideration regarding parameterization is: will the parameters determined by the inversion produce the desired information from the data? For example, if we wish to find the chargeability of a rock
Interpretation of complex resistivity and dielectric data

sample from amplitude and phase measurements of the complex resistivity, then we should formulate amplitude and phase in terms of chargeability, and not in terms of other related parameters, such as resistances in an equivalent circuit.

Another important consideration is to exclude solutions which are physically meaningless. Since negative resistivity has no meaning, it is expedient to use $\log R$ instead of $R$ as the parameter specifying the resistivity at zero frequency. As a result, negative resistivities cannot in any way be included in an estimate of the parameter confidence interval for $R$.

The third important consideration concerns linearization of the problem. All of the relaxation models which we have discussed, except for the simple resistance, have observables such as amplitude and phase which are non-linear functions of the model parameters. However, our ridge regression algorithm utilizes only the first derivative of the data with respect to the parameters. Thus it makes the implicit assumption that the relation between each observable and every model parameter is approximately linear. How good this assumption is will affect the speed of the convergence and the accuracy of estimates for the uncertainty in the determination of each parameter. We will attempt to illustrate later with an example, how different parameterization can result in extremely poor and extremely good linear approximations for the true behavior of the observable.

Data error and weighting

For the inversion algorithm given by (88) we have tacitly assumed that no weighting of the data was required since all the data were equally influenced by noise. For the simultaneous inversion of multiple data sets or inversion of data containing a few bad points, this assumption is no longer valid and we must first multiply the unweighted derivative matrix, $B$, and unweighted data difference vector, $A_g$, by a weight matrix, $W$, in order to obtain satisfactory results:

$$\mathbf{A} = \mathbf{W} \mathbf{B}$$  \hspace{1cm} (90)

$$\Delta y = \mathbf{W} \Delta g$$  \hspace{1cm} (91)

If the error in each measurement is independent of the error in the other measurements, as is usually assumed, then $W$ reduces to a diagonal matrix with entries which are inversely proportional to the standard deviation of each data point:

$$W_{ii} = \frac{1}{\sigma_i}$$  \hspace{1cm} (92)

We could possibly estimate $\sigma_i$ by making a large number of repeat measurements at each point, however this would be rather time-consuming. It is much
simpler to assign relative weights to the data and then later obtain an estimate for the data error from the reduced chi-square statistic,

\[ \chi^2 = \frac{\Delta y^T \Delta y}{n - m}, \]  

(93)

where \( n \) is the number of data points and \( m \) is the number of parameters. As discussed by Bevington [1969], \( x^2 \) will be the best estimate for the data variance at those data points associated with weights of 1.0 if the relative weights were indeed correct, the theoretical model was appropriate, and the errors were normally distributed.

Obviously the easiest relative weights to assign are weights which all have the same value for each data set. It is thus expedient to choose as observables those quantities which have roughly constant measurement error at all frequencies. This is one main consideration behind our choice of log amplitude as a common observable. For measurement of both complex resistivity and complex permittivity, the error in amplitude is very often a relatively constant percentage of the amplitude value. Thus if we chose amplitude and not log amplitude as the observable we would be required to specify weights which are inversely proportional to the magnitude of each measurement. Choice of log amplitude as the observable is possible, because amplitude can never be negative, and is desirable in that it eliminates this weight requirement.

Although easy specification of weights is certainly one consideration in the total problem formulation involving choice of both parameters and observables, we must also remain aware that the selection of observables will affect the linearization of the problem just as much as the selection of parameters. In our later example of linearization we change both parameters and observables in order to illustrate good and bad linear approximations.

Our emphasis so far in problem formulation has been on the choice of parameters (either relaxation parameters or equivalent circuit parameters) and on the proper choice of observables: either real and imaginary, amplitude and phase or amplitude and loss tangent, or perhaps logarithmic instead of linear, or possibly reciprocal as opposed to normal (conductivity instead of resistivity). All these different formulations are possible after we have decided on a specific relaxation model. The question remains: how do we choose this model in the first place? The most straightforward approach would be to base our model on the physical processes taking place; however, we may be attempting to determine the dominant process influencing our data out of several possible mechanisms. In such cases it is sometimes useful to employ the reduced chi-square as a goodness-of-fit criterion. We note that since the reduced chi-square is obtained by dividing the square error \( \Delta y^T \Delta y \) by \( n - m \), the smallest reduced chi-square will be obtained by the model which provides the best fit with the fewest parameters. This is commensurate with the philosophy that if we do not know exactly the physical process, the best model is the simplest model which still fits the data.
Interpretation of complex resistivity and dielectric data

Information density

If we wish to find out which data points are contributing the most information toward the determination of our model, one method would be to examine the $A$ matrix. Large positive or negative values will indicate which data points are important in determining each of the various parameters. For an estimate of which data points are important in the resolution of the total model, however, it is often convenient to examine the information density matrix given by

$$ S = AH $$

where

$$ H = (A^T A)^{-1} A^T $$

is the generalized inverse. We find that $S$ is a symmetric, diagonally dominant matrix with dimensions $n \times n$. Thus an approximate idea of which data points are the most important may be obtained from examination of the diagonal elements of $S$. This also provides a useful check on our original choice of weights for the problem. For example, if we expect that two different data sets should be providing roughly equal information, but find that the diagonal elements of $S$ are an order of magnitude smaller for one data set than for the other, our choice of weights may be seriously in error.

Parameter statistics

Once we have reached convergence and obtained a set of parameters which provide the best least-squares fit to our data, the next most important requirement is to obtain some idea of the uncertainty in the parameters. If our problem is very poorly posed we may find that $A^T A$ still remains singular even at convergence. In such cases it is necessary to maintain a non-zero value of $\lambda$ merely to eliminate the zero eigenvalues and thus permit inversion of the matrix. A very approximate idea of which parameters have high uncertainty may then obtained by examining the diagonal elements of the resolution matrix,

$$ R = HA $$

where $H$ is now given by

$$ H = (A^T A + \lambda I)^{-1} A^T $$

Those diagonal elements of $R$ which are significantly less than 1.0 indicate parameters which are very poorly resolved [Angoran 1975].

Usually, however, it is more desirable to deal with models which are not overly complex and to acquire sufficient data to determine adequately the model parameters. In such cases $A^T A$ will not be singular at convergence and the resulting resolution matrix, with $\lambda$ set equal to zero, will be very nearly equal
to the identity matrix (within numerical roundoff). For these well-posed, over-determined problems a more accurate estimate of the uncertainty in each parameter may be obtained by examining the parameter covariance matrix, given by

$$\text{cov}(\mathbf{p}) = \chi_r^2 (\mathbf{A}^T \mathbf{A})^{-1}$$  \hspace{1cm} (98)

The best estimate of variance for each parameter may be simply obtained from the diagonal elements of \(\text{cov}(\mathbf{p})\). We may also obtain, by appropriate normalization of \(\text{cov}(\mathbf{p})\), the parameter correlation coefficients,

$$\text{cor} p_{ij} = \frac{\text{cov} p_{ij}}{\sqrt{\text{cov} p_{ii} \cdot \text{cov} p_{jj}}}$$  \hspace{1cm} (99)

Correlation coefficients near 1.0 indicate that only the difference of two parameters is well-resolved, whereas correlation coefficients near -1.0 indicate that only their sum is well-resolved. If the inversion is carried out with respect to log parameters (as we have done in the remainder of this paper), correlation coefficients of +1.0 and -1.0 indicate that only the quotient or product of two parameters is well-determined.

8. Applications

There are a large number of areas where inversion can be very usefully applied in the analysis of complex resistivity and dielectric data. It will not be possible to cover all of these areas in this paper, or even to discuss in rigorous detail all the relevant statistical information for the few examples that we have chosen. Our approach, instead, will be to focus on small segments of the interpretation of a fairly broad range of data. In this way we may perhaps indicate the variety of rock property problems that can be attacked with inversion and the usefulness of the technique.

Model selection

Our first application of inversion to analyzing the electrical properties of rocks will concern selection of the simplest relaxation model fitting given complex resistivity or dielectric data. *Bingham Granite Stock.* Shown in Figure 12a normalized amplitude data from laboratory measurements of the complex resistivity of a mineralized sample of the Bingham stock. The data were kindly provided by Nelson [1975]. We have attempted to fit five models of the Cole—Cole family to the data. The model which provides the lowest value of the reduced chi-square, shown in Figure 12b,
is No. 5, which corresponds to the Madden—Cantwell relaxation model (Table 1 in Part one). It is evident that increasing the complexity of the model to the Cole—Cole relaxation (No. 8) and to the generalized Cole—Cole relaxation (No. 10), does not appreciably improve the data fit. The only result is to increase the number of model parameters, thus decreasing the degrees of freedom, $n-m$, and consequently increasing the reduced chi-square. From this simple analysis we might conclude that out of the five models considered, the Madden—Cantwell model is the most appropriate model for the data.

Fig. 12. (a) Inversion of $|\phi(\omega)|$ obtained from laboratory measurements of a sample of the Bingham granite stock. The circles represent the observed data; the solid line represents the best-fitting relaxation model (No. 5).

(b) Goodness-of-fit values for the five relaxation models which were used to fit the data in (a).

12. ábra. (a) A Bingham gránítomzsból vett mintán végzett laboratóriumi mérések $|\phi(\omega)|$-jának inverziója. A körök jelentik a mért adatokat, a folytonos vonal jelenti a legjobban illeszkedő relaxációs modellt (5. számú)

(b) Öt, az (a) ábra adataihoz illesztett relaxációs modell illesztés-jósági értéke

A further comparison of the data and the best-fitting Madden—Cantwell model, shown as the solid curve in Figure 12a, tends to confirm our conclusion. There is no obvious bias; any difference between the model and the data appears to be random measurement error. This brings us to the subject of a recent controversy [NELSON and VAN VOORHIS 1977; ZONGE and WYNN 1977]. We do not plan to take sides in this controversy; however, we do wish to illustrate how
inversion might be used to evaluate the accuracy of rock electrical measurements. The value for the reduced chi-square obtained for model No. 5 in Figure 12b was $1.42 \cdot 10^{-6}$. Since the theoretical model adequately fits the data without apparent bias, we might assume that this value provides a reasonable estimate for the true variance of the data. Our resulting conclusion is that the amplitude measurements, shown plotted on a double logarithmic scale in Figure 12a (vertical exaggeration $10:1$), were made with an average accuracy of $0.27\%$ over the four and one-half decades of frequency. In contrast, the total frequency effect over this range was $14\%$.

**Lunar Rock 73,275–8 105 °C** We continue with the subject of model selection by examining the phase angle data from measurement of the dielectric constant of a lunar rock shown in Figure 13a. We again attempted to fit several members of the Cole—Cole family of models to the data. In this case, the outcome, shown in Figure 13b, was rather different. The model which provided the lowest value of the reduced chi-square was the most complex of the series. The results suggest that the phase data is significantly non-symmetric and consequently is not well-fitted by the simple Cole—Cole model or by special cases of this model.

**Fig. 13.** (a) Inversion of phase data obtained from measurement of the dielectric constant of lunar rock 73 275–8 at 105 °C. The circles represent the observed data; the solid line represents the best-fitting relaxation model (no. 10)

(b) Goodness-of-fit values for the five relaxation models which were used to fit the data in (a)
Iron Mountain 5 Our next example illustrating choice of an appropriate relaxation model concerns in-situ complex resistivity data from an open pit magnetite mine near Iron Springs, Utah. Both amplitude and phase data were obtained and were inverted simultaneously to produce the resulting best-fitting Cole—Cole model shown in Figure 14. Also shown in the figure are the estimated parameter standard deviations. Since our problem formulation involved log parameters as well as log data, the estimated parameter standard deviations are precisely displayed as percentages.

![Figure 14](image)

Fig. 14. Inversion of in-situ complex resistivity data obtained over massive powdery magnetite mineralization near Iron Springs, Utah. The observed data are represented by circles and squares; the solid lines indicate the best-fitting Cole—Cole complex resistivity model.

We mentioned previously that appropriate problem formulation requires consideration of several factors, not the least of which is the requirement that the resulting problem be adequately linear. We will now attempt to illustrate with the Iron Mountain 5 model and data, how two different problem formulations can lead to two vastly different linear approximations.
Shown in Figure 15 are three curves passing through the phase angle measurement made at 17.8 Hz. The curve labelled “A” corresponds to a problem formulation involving linear phase (not logarithmic) and linear parameters. Thus each of the columns of the derivative matrix will be composed of simple first order derivatives of phase with respect to each of the model parameters: $R$, $m$, $\tau$, and $c$. If we select $\tau$ as our parameter of interest, the single element of the derivative matrix, corresponding to the $\tau$ parameter and the 17.8 Hz frequency measurement, attempts to predict the behavior of phase as a function of $\tau$ by the formula,

$$\Phi(\tau) = \Phi_0 + \frac{\partial \Phi}{\partial \tau} (\tau - \tau_0),$$  \hspace{1cm} (100)$$

where $\Phi_0$ is 48.2 milliradians, $\tau_0$ is $5.74 \cdot 10^{-5}$ seconds, and the derivative is 425.9 milliradians per second. As shown in Figure 15 this formula is a very poor approximation to the actual curve for $\Phi(\tau)$ except in the region immediately surrounding $\tau = \tau_0$.

A much better approximation of the true $\Phi(\tau)$ behavior can be made if we
formulate the problem in terms of log $\Phi$ and log $\tau$. Since the appropriate element of the derivative matrix is 0.51, the approximation for $\Phi(\tau)$ on a double logarithmic scale becomes simply a straight line through $(\Phi_0, \tau_0)$ with slope 0.51. This approximation (the “B” curve in Figure 15) is very good over a five decade range of $\tau$, and thus suggests that this formulation will result in rapid convergence from a poor initial guess, and accurate prediction of the parameter confidence interval.

**Tyrone**

Our last example of simple model selection and fitting involves in-situ complex resistivity data from the Tyrone porphyry copper deposit near Silver City, New Mexico. Two main dispersions are very evident in the phase angle data shown in *Figure 16*. In order to fit the data we chose a transfer function formed by the multiplication of two Cole—Cole models.

![Inversion of in situ complex resistivity data obtained over the Tyrone porphyry copper deposit](image_url)

*Fig. 16. Inversion of in situ complex resistivity data obtained over the Tyrone porphyry copper deposit*

*16. ábra. A Tyrone porfiros rézérc előforduláson kapott in situ komplex ellenállás adatok inverziója*

*Рис. 16. Инверсия комплексных данных сопротивления, полученных в условиях естественного залегания на месторождении порфирородной медной руды Тайрон.*
During the inversion process we encountered difficulty at the lower frequencies. Although the phase angle measurements in this region describe a smooth curve, and thus appear relatively accurate, there is only a slight suggestion of what the low frequency phase behavior might be. This slight character in the curve is actually below the noise level of the phase angle measurements made at higher frequencies. As a result, when an attempt was made to fit the \( \log \Phi \) data without using weights, the inversion routine treated the slight bend in the curve at low frequencies as noise, and produced a poor data fit with resulting high parameter standard deviations.

In order to produce a more acceptable fit to the data, we effectively introduced our own bias into the inversion through our selection of relative weights. Use of larger weights at the lower frequencies forced a more precise fit to the data in this region and substantially reduced the estimated uncertainty in the parameters describing the low frequency phase angle peak. The actual credibility of these new small parameter standard deviations is, however, directly dependent on our assumption that the low frequency measurements have a substantially lower noise level than those at higher frequencies. The estimated uncertainty in the parameters, \( \tau_2 \) and \( c_2 \), describing the high frequency dispersion were left relatively unchanged by the new weights.

**Identification of physical processes**

Often the ultimate goal of our attempts to fit relaxation models to complex resistivity and dielectric data is really to try to determine, and then characterize, the poorly-understood physical processes taking place. 

*Pyrite electrode—electrolyte interface* In our first example we will try to examine the basic mechanism responsible for the complex resistivity behavior of mineralized rocks. Shown in Figure 17 is an equivalent circuit which is quite often used to model the resistive, capacitive, and diffusive behavior of the semiconductor—electrolyte interface [Ward and Fraser 1967]. Also shown in Figure 17 are values of the circuit components obtained from inversion of amplitude and phase measurements made on pyrite electrodes over seven decades of frequency [Klein and Pelton 1976]. In Figure 18 we show the observed amplitude and phase data, and the theoretical curves obtained from inversion. The agreement is excellent. This suggests that the simple circuit in Figure 17 adequately describes the impedance of the pyrite-electrolyte interface, and that we thus may evaluate the relative importance of the different conduction processes by more closely examining the values of the circuit components. One conclusion is that, for pyrite electrode data shown in Figure 18, the reaction resistance is an extremely minor component of the total electrode impedance. It is essentially undetectable relative to the other components of the equivalent circuit. Another conclusion is that the double layer capacitance is also quite small (a few microfarads per square centimeter) and therefore does not provide appreciable conduction across the interface at frequencies much below 1 kHz.
By allowing the exponents of the Warburg element and the double layer capacitance to vary we were able to model effectively non-ideal components; however, we found that the exponent $c_1$ was very close to that of the ideal Warburg ($c = 0.5$) and that the exponent $c_2$ was quite close to that of ideal capacitive behavior ($c = 1.0$).

The higher frequency dependence of the capacitive behavior of the interface is strikingly displayed in Figure 18 by the steep high frequency asymptotic slope of the phase curve.

Scott—West Artificial Rocks We devoted considerable discussion in the first part of this paper toward attempting to define and separate complex resistivity and complex dielectric phenomena. We will now illustrate with an example how inversion might be used to achieve this separation. The data shown in Figure
Fig. 18. Amplitude and phase spectra obtained from measurement of the impedance of two pyrite electrodes

18. ábra. Két pirít elektróda impedanciájának méréséből kapott amplitúdó és fázis spektrum

Puc. 18. Спектр амплитуд и фаз, полученный по измерению импеданса двух пиритовых электродов.

19 and Figure 20 were obtained from complex resistivity measurements of artificial rock samples prepared by Scott and West [1965] from cement, quartz and pyrite. Due to the high pressures applied during preparation, the samples have very low porosity and consequently, very high resistivity. Thus for at least the highest frequencies, we might expect a substantial decrease in the measured complex resistivity, due to dielectric conduction phenomena.

Sample 124B contains no pyrite mineralization and exhibits relatively simple spectra. By casual comparison of the theoretical phase angle curves in Figure 5 with the results shown in Figure 19, it appears that a single Cole—Cole resistivity dispersion might adequately fit the data. This was attempted; however it was found that the theoretical high frequency phase angle curve was very definitely lower than the observed data. Another possible model, which gave a
Interpretation of complex resistivity and dielectric data

**Fig. 19.** Inversion of complex resistivity spectra obtained from measurement of Scott-West sample 124B (no pyrite)

19. ábra. A 124B Scott-West mintán (pirít nélküli) végzett mérésekből kapott komplex ellenállás spektrumok inverziója

**Fig. 20.** Inversion of complex resistivity spectra obtained from measurement of Scott-West sample 94B (containing 2% pyrite)

20. ábra. A 94B Scott-West mintán (2% pirít tartalom) végzett mérésekből kapott komplex ellenállás spektrumok inverziója

**Fig. 19.** Inversion of complex resistivity spectra obtained from measurement of Scott-West sample 124B (no pyrite)

19. ábra. A 124B Scott-West mintán (pirít nélküli) végzett mérésekből kapott komplex ellenállás spektrumok inverziója

**Fig. 20.** Inversion of complex resistivity spectra obtained from measurement of Scott-West sample 94B (containing 2% pyrite)

20. ábra. A 94B Scott-West mintán (2% pirít tartalom) végzett mérésekből kapott komplex ellenállás spektrumok inverziója
better fit to the data, included a constant resistivity with a single Cole—Cole dielectric dispersion. However, this model requires an unreasonably high value of the dielectric constant at the lower frequencies (in the range of several thousand). The third model which we tried gave the best fit to the data and is shown in Figure 19. We have assumed a single Cole—Cole resistivity dispersion and an entirely real dielectric constant (which was subsequently determined to be 45).

Our conclusion from this exercise is that the major physical process taking place is a complex resistivity relaxation, although dielectric conduction does become important at the highest frequencies. Since there was no metallic mineralization in the sample, we cannot attribute the Cole—Cole resistivity dispersion to metallic electrode polarization.

Shown in Figure 20 are the results from measurements of the complex resistivity of sample 94B which contained 2% pyrite. In addition to the presumed dielectric and membrane polarization behavior at high frequency, we also have a prominent low frequency dispersion apparently caused by the pyrite. The time constant of this relaxation is much longer than the time constants obtained for pyrite mineralization in other artificial rocks [GRISSEMAN 1971, SILL and DEWITT 1976]. This is most likely due to the abnormally high D.C. resistivity of the Scott-West samples (notice the equations for the time constant in Figure 2; τ is strongly dependent on the D.C. resistivity for each of the three equivalent circuits).

As a final note on these artificial rocks we have plotted in Figure 21 the diagonal terms of the information density matrix corresponding to the data from sample 94B. Our formulation of the problem was in terms of log phase, log amplitude and log parameters. However, since we did not specify different relative weights between the phase and amplitude data sets, by default we made
the assumption that the percentage error in the phase measurements was roughly equal to the percentage error in the amplitude measurements. Since this assumption is really quite debatable, it was useful to check the information density matrix and confirm that the values for the two data sets were at least not different by several orders of magnitude. We note that the information density is quite low for those amplitude data which are flat and featureless, but that the values strongly increase at the higher frequencies where the amplitude exhibits strong curvature and the phase becomes flat and featureless. The two data sets thus tend to complement each other in providing useful information over the total frequency range.

Roosevelt KGRA We have noted what appears to be very pronounced effects due to membrane polarization in the Scott-West rocks. Lest the reader obtain the impression that these effects occur only under unusual circumstances in artificial rocks, we present in Figures 22 and 23 results obtained from complex resistivity measurements of drill cores from the Roosevelt Hot Springs geothermal area [TRIPP 1976].
The phase angle results shown in Figure 22 correlate with a very highly altered section of the drill log where the rock was 58% kaolinite. The cation exchange capacity per unit pore volume ($Q_v$) is large and the sample exhibits a prominent complex resistivity dispersion at the higher frequencies, in spite of the fact that no metallic-conducting minerals were observed present. In order to fit the data we have assumed that conduction through the clays acts in parallel with the normal ionic conduction through pore passages in the rock. The result is an equivalent circuit representation which is the same as that shown earlier in Figure 2a. Although the circuit is known to produce Cole—Cole relaxational behavior we have inverted for the components of the equivalent circuit rather than the Cole—Cole model parameters.
These circuit components may be more readily compared with those fitting the data for the second drill core sample shown in Figure 23, where we have considered a more complex equivalent circuit than that giving rise to a single Cole—Cole relaxation. This second sample exhibited little clay alteration but showed prominent iron staining, suggesting the presence of pyrite. In order to fit the two indicated dispersions in the phase angle spectra we have utilized the equivalent circuit shown in Figure 24, which has two frequency-dependent paths. In total, there are three parallel current paths, by which we have attempted to simulate three conduction mechanisms: conduction through the normal unblocked rock pores, through clay blocked pores and through pyrite blocked pores. Since the phase angle data shows no indication of rolling over at high frequency, we are unable to determine any value for $R_z$; the other parameters, however, are all reasonably well determined. The relatively low values of the two frequency dependences ($c_1$ and $c_2$ in Figure 23) suggests diffusion phenomena rather than capacitive or inductive behavior.

![Equivalent Circuit for Rocks Containing Both Pyrite Mineralization and Clay Alteration](image)

*Fig. 24. Equivalent circuit for rocks containing both pyrite mineralization and clay alteration*

*24. ábra. Pirites ércesedést és agyagos elváltozást is mutató közetek ekvivalens áramköré*

*Рис. 24. Эквивалентная электрическая схема пород, показывающих как пиритовое оруденение, так и глинистое изменение.*
Inductive coupling recognition The last topic under this general discussion of recognition of physical processes, we will only briefly mention since it has been discussed elsewhere [Pelton et al. 1978]. But basically, inductive coupling response is perhaps the easiest process to identify in complex resistivity observations. We have noted that metallic electrode polarization and membrane polarization both result from diffusion phenomena, thus the only hope of distinguishing between the two rests on perhaps indentifying statistically significant differences in the time constant and chargeability (field experience and our limited laboratory measurements suggest that membrane polarization has a relatively short time constant). There are similar difficulties in distinguishing between dielectric effects and ionic conduction processes in resistive laboratory samples. Dielectric conduction in the Scott-West rocks was manifested by only a slight increase in phase at the very highest frequencies.

The effects produced by inductive electromagnetic coupling on complex resistivity data are by comparison relatively easy to detect. All solutions for inductive coupling so far [Millet 1967, Hohmann 1973], indicate that the frequency dependence for coupling response is very close to 1.0. This is consistent with the observation that many EM problems can be cast in terms of an induction number which involves frequency to the first power. In contrast, the frequency dependence of the complex resistivity response arising from natural earth materials is very low, typically about 0.25. Thus inductive coupling effects should be readily recognizable by a prominent increase (or decrease, depending on whether the coupling is positive or negative) in the slope of the phase angle response on a double logarithmic plot.

Quantitative evaluation of spectra

Our analysis thus far has been mainly concerned with identification of models and physical processes. However, perhaps the most immediate, useful application of our inversion routine will be in the quantitative evaluation of spectra.

Lunar Rock 73275–8 27 °C In Figure 25 we show complex dielectric data obtained from measurement of the same lunar rock for which data was presented earlier in Figure 13. This latter data set was acquired with the rock temperature held constant at 27 °C instead of 105 °C. Again we have fitted a generalized Cole—Cole model to the data so that the new parameter values may be usefully compared with the previous parameter values obtained at 105 °C. If we had several sets of data at intermediate temperatures it would be possible to identify and trace relatively subtle variations in the relaxation parameters with temperature. This may be useful in regard to two objectives: first, to provide desired engineering information on the dielectric constant as a continuous function of temperature, and second, to use the known variation with temperature to identify more accurately the physical processes responsible for the dielectric behavior.
Interpretation of complex resistivity and dielectric data

Another area where engineering information is badly needed is the complex resistivity measurement of mineralized rocks. Although accurate IP measurements have been made for more than twenty years, we still have very little precise, reliable data on the variation in IP response due to mineral concentration and grain size.

Shown in Figure 26 is an example of some of the data which has been collected. The plot shown is of real conductivity normalized by the real conductivity at $10^4$ Hz. The data were acquired by Grissemann [1971] during a study of artificial rocks composed of cement, quartz and pyrite in varying concentrations and grain sizes. In order to interpret the results, we assumed a Cole—Cole model for the complex resistivity, then took the logarithm of $\text{Re}[1/\sigma(\omega)]$ as our observable. The fit to the data, shown in Figure 26, is remarkably good. The

**Grissemann data** Another area where engineering information is badly needed is the complex resistivity measurement of mineralized rocks. Although accurate IP measurements have been made for more than twenty years, we still have very little precise, reliable data on the variation in IP response due to mineral concentration and grain size.

Shown in Figure 26 is an example of some of the data which has been collected. The plot shown is of real conductivity normalized by the real conductivity at $10^4$ Hz. The data were acquired by Grissemann [1971] during a study of artificial rocks composed of cement, quartz and pyrite in varying concentrations and grain sizes. In order to interpret the results, we assumed a Cole—Cole model for the complex resistivity, then took the logarithm of $\text{Re}[1/\sigma(\omega)]$ as our observable. The fit to the data, shown in Figure 26, is remarkably good. The

![Lunar Rock Diagram](image_url)
average data error is only 0.36%, yet the conductivity varies by 300% over the four decades of frequency. Since the data error is very low, the parameters for the Cole—Cole resistivity relaxation are very precisely determined, as indicated by the small standard deviations shown in the figure.

Fig. 26. Inversion of real conductivity data obtained by Grisseman [1971] from measurement of artificial rock sample 84B

We were able to fit simple Cole—Cole models to all of the artificial rock real conductivity data obtained by Grisseman. It was then possible to plot the variation in Cole—Cole parameters with pyrite grain size and concentration. Shown in Figure 27 is one such plot. The slope of 2.0 on the double logarithmic plot indicates that the time constant is proportional to the square of the pyrite grain size.

Once the Cole—Cole model parameters are obtained from inversion of a data set, it is a simple matter to construct a theoretical curve for any desired observable. For example, if we achieve a good fit to real resistivity data, we may immediately calculate the imaginary resistivity curve by substituting the determined parameters into the analytical expression for the Cole—Cole transfer function and taking the imaginary part. This is one reason why we only briefly...
mentioned, in the first part of this paper, the Hilbert transform relation between the real and imaginary parts of a causal transfer function. In theory, the relation is simple, but in practice it is quite difficult to perform a Hilbert transform numerically on noisy observed data and obtain meaningful results [WILSON 1977].

\[ T = 0.4 \, \text{g}^2 \]

Fig. 27. Time constant versus pyrite grain size obtained from inversion of data gathered by COLLETT [1959], GRISSEMAN [1971], and DEWITT and SILL [1976]


Рис. 27. Функция постоянной времени — размер зерни пирита по инверсии данных, полученных Колетом (1959 г.) Грисманом (1971 г.), а также ДэВитом и Силлом (1976 г.)
We will now use the Cole—Cole parameters obtained from the inversion in Figure 26, and the Hilbert transform relations inherent in our causal relaxation model, to check Grisseman’s results. Shown is Figure 28 is a plot of the tangent of the phase angle, determined from our Cole—Cole parameters, along with actual observations of the tangent of the phase angle reported by Grisseman [1971]. The two curves are in reasonable, but no perfect agreement. The authors are here indebted to Wong [1977] for bringing to our attention the correct logarithmic scale for Grisseman’s original data. Our first analysis gave very poor agreement, indicating that the observations were not causal, and that therefore time invariance or linearity were not maintained, or that there was some problem with the measuring apparatus or data reduction.

Examples of measurements which obey the causality requirements almost perfectly are shown in Figure 18 and 19. The solid curves for amplitude and phase were calculated from the same model parameters.

![Figure 28](image)

**Fig. 28.** $\tan \Phi$ observed by Grisseman [1971] compared with $\tan \Phi$ calculated from Cole—Cole model parameters obtained in Figure 26.

28. ábra. A Grisseman által mért (1971) $\tan \Phi$ értékek összehasonlítása a 26. ábrán kapott Cole—Cole modell paramétereiből kapott $\tan \Phi$ értékekkel

Рис. 28. Сопоставление значений $\tan \Phi$, измеренных Грисманом (1971 г.) с значениями $\tan \Phi$, полученными по параметрам модели Кол-Кол на рис. 26.
Time domain data All of our examples of interpretation of complex resistivity and dielectric data have so far concerned measurements made only in the frequency domain. We mentioned in the first part of this paper that frequency domain measurements tend to be easier to make, and to interpret, than time domain measurements, particularly when information is desired over several decades of time or frequency.

However, to illustrate how similar complex resistivity and dielectric interpretation might be carried out in the time domain we will discuss some decay curves published by Madden and Cantwell [1967]. Shown in Figure 29 are decay data from artificial rocks originally obtained by Collett [1959] but replotted by Madden and Cantwell [1967]. The dashed curve in the figure was calculated by Madden and Cantwell [1967] from their approximate derivation of the IP decay, corrected for the 1 second switching time used by Collett [1959]. Since the calculated curve fits only the data for smaller grain sizes, Madden and Cantwell suggest that Collett’s data for the larger grains was adversely affected by high current densities and the unusual pore geometry created in artificial rocks. We would like to offer here a considerably different interpretation for change in decay shape with increasing grain size, and at the same time to illustrate the various steps required in the interpretation of time domain data.

![Graph](image_url)

**Fig. 29.** Decay data obtained by Collett [1959] from measurement of artificial rocks which were replotted by Madden and Cantwell [1967], compared with theoretical curves.
In our analysis of Grisseman's frequency domain data (Figure 26) we obtained a very good fit to his real conductivity measurements using a Cole—Cole complex resistivity model with frequency dependence very nearly equal to 0.5. This value ranged between 0.45 and 0.55 for almost all of his data on artificial rocks.

Good fits to Cole—Cole models with values of $c$ near 0.5 were also observed for our measurements of the Scott-West artificial rocks (Figure 20) and for other recent measurements of artificial rocks at our laboratory [Sill and DeWitt 1976]. It is thus our tentative conclusion that where a very restricted range of grain sizes is involved, as in most artificial rocks, the relaxation behavior closely follows that of a Cole—Cole model with the frequency dependence approximately equal to 0.5 (our "Warburg" model in Table 1). For natural mineralized rocks where the range of grain sizes is typically broader, the dispersion is broader, and the value of the frequency dependence is typically near 0.25, although it ranges from 0.5 to as low as 0.125 [Pelton et al. 1978].

Thus in order to interpret the time domain decay curves shown in Figure 29, where the grain size in each sample was rigidly controlled, we decided to adopt the Warburg relaxation model. In the first part of this paper we derived the exact negative step function response for this model and then corrected for two common transmitter switching waveforms. The results, shown in Figures 8a, and 8b, were two sets of curves, whose decay shape depends on only one variable, the ratio of the relaxation time constant to the transmitter pulse length. We have displayed four of these theoretical decay curves as solid lines in Figure 29. Since the transmitter pulse length was 1 second, they correspond to different relaxation time constants ranging from 0.01 seconds to 10 seconds. It is now apparent from Figure 29, that the strange, systematic change in decay shape, noted by Madden and Cantwell [1967], can be very simply explained by an increase in time constant as a function of grain size, without resorting to nonlinear behavior or unusual pore geometry.

As an exercise, to compare the trends observed in Collett's data, Grisseman’s data and data from DeWitt and Sill [1976], we plotted all three sets of results in Figure 27. The similarity of trends is remarkable, considering that absolutely no effort was made to create the same porosity in the different sets of samples, or to use similar electrolyte concentrations.

Madden and Cantwell [1967] also published nine other decay curves for natural mineralized rocks. We have plotted only two of these data sets in Figure 30 since the other seven decay curves fall between these two. The measurements were made using sequential switching with a transmitter pulse length of 7 seconds. The solid lines shown in the figure are two of our theoretically calculated curves based on a Warburg model and corrected for sequential switching. It is apparent that the two different curve shapes might possibly be explained by a factor of 10 difference in the time constant.

We would like to be more thorough here, and to use our ridge regression inversion routine to solve in the time domain for all the Cole—Cole model parameters, including the frequency dependence, $c$; rather than assuming a
value of $c$ and then superimposing type curves to estimate the time constant. This can certainly be easily accomplished, and would be desirable, if more and better quality data in the time domain become available.

![Graph](image)

**Fig. 30.** Decay data obtained by Madden and Cantwell [1967] from measurements of natural rocks, compared with theoretical curves.

**Transformation between domains**

The final subject we will consider, involves numerical transformation from one set of measurements to another; notably from the frequency domain to the time domain. First, however, we will discuss transformation between the two types of common measurements made in the frequency domain.

**Phase versus frequency effect** If we were to assume a constant phase model for complex resistivity (such as the Drake model when $\omega \tau > 1$), it would be very easy to establish a universal relationship between frequency effect and phase. The frequency effect measurement,

\[
FE_{T-h} = \frac{|q(l)| - |q(h)|}{|q(h)|}
\]  

(101)

can be considered as merely an attempt to specify the slope of $|q(\omega)|$ on a double logarithmic plot, by taking the first backward difference. The true slope, for the Drake model with parameters $R$, $\tau$ and $a$, when $\omega \tau \gg 1$ is simply
FE_T = \frac{a}{t} \text{ decades/decade} \\
\cong 100 \frac{ae^\%}{\text{decade}} \quad (102)

for \( a < 1 \) and \( e = 2.71828 \ldots \) The phase, \( \Phi \), is also readily given in terms of \( a \) by

\[
\Phi = a \frac{\pi}{2} \text{ radians}, \\
= 500a\pi \text{ milliradians} \quad (103)
\]

[VAN VOORHIS et al, 1973]. Thus we have the very simple relationship

\[
\Phi \cong \alpha \cdot \text{PFE}_T 
\]

where

\[
\alpha = 5 \frac{\pi}{e} \text{ milliradians} \\
= 5.8 \text{ milliradians} \\
= 0.33 \text{ degrees} \quad (105)
\]

and \( \text{PFE}_T \) represents percent frequency effect per decade.

We tested this theoretical relation by plotting \( \Phi_{0.1\text{Hz}} \) against \( \text{PFE}_{0.1-1.0\text{Hz}} \) for in-situ data obtained by PELTON et al. [1977]. Each black dot in Figure 31 is essentially free of measurement error since we first inverted the spectra, and then used the theoretical relaxation model to calculate \( \Phi_{0.1\text{Hz}} \) and \( \text{PFE}_{0.1-1.0\text{Hz}} \). Thus the scatter in the diagram arises solely from the fact that the in-situ spectra were not constant phase, and thus all calculated \( \alpha \) were slightly different from 0.33 degrees.

To obtain an average value for \( \alpha \) we conducted a simple linear regression in double logarithmic space, and then converted the estimated percentage standard deviation for \( \alpha \) to a linear standard deviation. Shown in Figure 31, along with our least-squares fit (solid line; \( \alpha = 0.35 \pm 0.02 \) degrees) are dashed lines corresponding to other relations obtained by ZONGE [1972] and SCOTT [1971]. Scott graphically derived several different values for \( \alpha \) ranging from 0.30 degrees to 0.35 degrees to 0.43 degrees, whereas Zonge used a different theoretical approach than ours to obtain a value of \( \alpha = 0.31 \) degrees.

The main purpose of this exercise was to illustrate that a reasonably accurate linear relation between PFE and phase can be derived extremely easily using the Drake model. Substantial differences from this theoretical relation indicate that the phase is not constant. A few rapid calculations using PFE and phase obtained from a Cole—Cole model reveal that large values of \( \alpha \) correspond with large time constants or decreasing phase with frequency, whereas small values of \( \alpha \) correspond with small time constants or increasing phase. The scatter diagram of Figure 31 roughly indicates how much variation might be expected in noise-free measurements of \( \alpha \) over natural mineralization.
Frequency domain—time domain conversion

By now the reader is probably all too well aware of our fascination with the Cole—Cole model. The reason for this preoccupation is that the model is extremely general, to the extent that it can fit virtually any relaxation transfer function with either one or a combination of several of these models. (Remember that the Debye model, which corresponds to a delta function representation in the distribution domain, is just a special case of the Cole—Cole model.) Yet in spite of this generality, the model has a very simple mathematical form which is easily programmable in both the frequency domain and the time domain.

We have already examined two sets of time domain data, and attempted to fit a special case of the Cole—Cole model ($c = 0.5$) to the data. If we were able to obtain, with confidence, all of the Cole—Cole model parameters through analysis of the time domain data (obtaining $m$, for example, through measurement of $m_{\text{obs}}$, knowledge of the transmitter waveform, and use of Figure 9) we could then simply calculate any observable in the frequency domain, such as amplitude or phase, through the relation for $h(\omega)$ given by (21).
Similarly, we can use the parameters determined by inversion in the frequency domain, to calculate the time domain response. This exercise has been carried out with the parameters obtained from inversion of the Iron Mountain 5 in-situ data shown in Figure 14. The expressions (47) and (50) were programmed on an HP-67 pocket calculator and used to obtain the calculated step function decay curve shown in Figure 32.

To illustrate transformation of more difficult spectra, we re-examined the Tyrone 1 in-situ data shown in Figure 16. The frequency domain model which was originally fitted to the data was not very satisfactory since it was a multiplication of two Cole—Cole models which, in turn, corresponds to convolution of two Cole—Cole models in the time domain. Since convolution of two functions over seven decades was beyond the capability of our pocket calculator (or the University of Utah Univac 1108, for that matter) we decided to re-invert the spectral data to the following model:

$\phi(\omega) = R \left[ \frac{m}{1 + (i\omega \tau_1)^c_1} + \frac{1 - m}{1 + (i\omega \tau_2)^c_2} \right]. \quad (106)$

The parameters, $R$, $m$, $\tau_1$, and $c_1$ previously produced a good fit to the low frequency phase angle peak, so they were held constant. The new model parameters, $\tau_2$ and $c_2$ were determined to be
Interpretation of complex resistivity and dielectric data

\[ \tau_2 = 1.70 \cdot 10^{-8} \pm 14.8\% \]  
\[ c_2 = 0.360 \pm 2.4\% \]

and the resulting new “addition model” curve was found to be virtually identical to the old “multiplication model” curve shown as a solid line in Figure 16. However, since addition in the frequency domain corresponds to addition in the time domain (because the Fourier transform is linear) it was consequently a simple matter to add together two Cole—Cole decay curves to produce the “double decay” curve for (106) shown again in Figure 32.

We have plotted the decay for Tyrone 1 over fourteen decades of time, in order to illustrate its asymptotic behavior. However, this behavior is not really well known since we only have seven decades of frequency information \((10^{-1} < \omega < 10^6)\). Thus the decay could be a little faster or slower for \(10^1 < t < 10^{-6}\).

Also, both the decays assume an infinitely long charging time. If the transmitter pulse length is not very long compared to \(\tau_1\), the decay curves must be corrected for the effects of switching by adding the appropriate series of positive and negative step function responses.

The point we are trying to make in this section, is that transformation between the frequency domain and the time domain can be carried out relatively easily using inversion and sums of Cole—Cole models (or of any other relaxation models, such as the Cole—Davidson model, which have easily programmable response in both domains).

Partly because the time domain decay is so long and drawn out (Tyrone 1 takes roughly 10 decades to decay one decade), and partly because \(10^7\) point Fast Fourier Transforms are not readily available, it is not possible to transform broad-band data between the time and frequency domains directly, by using the FFT. A 1024 point FFT, for example, corresponds to less than three decades of information. An alternative, innovative method of transformation is described by Wilson [1977]. He first transforms to a logarithmic frequency scale then finds that the Hilbert transform and decay spectra relations become algebraic rather than integral equations. His method is advantageous in that transformation is not dependent on any particular relaxation model; however, extrapolating functions are required at high and low frequencies, and the transformation is adversely affected by noise in the observed spectra. In contrast, our method is model dependent, but has the advantage that no extrapolating functions are required at high and low frequency, since the asymptotic behavior of \(h(\omega)\) is automatically specified. In addition, the transformation is relatively immune to noise in the observed spectra. Although some noise does persist through to the time domain (in the form of parameter uncertainties) much of the measurement error is very effectively filtered out at the start, through the least squares fit to the data provided by the inversion.
9. Discussion and summary

This concludes our attempt at treating a reasonably broad subject: the analysis of complex resistivity and dielectric data. By our original choice of this topic we were almost predestined to commit a cardinal sin: the production of a research paper which is longer than the average reader's time and patience. However, the alternative was to produce several smaller, self-contained papers on special topics such as the forward problem, quantitative evaluation of spectra and transformation between the different domains. This would result in an unfortunate lack of integration and unnecessary redundancy.

In the single paper which we have written instead, we have attempted to summarize briefly the essential requirements of relaxation models for complex resistivity and dielectric behavior. This led to a consideration of the behavior in the frequency domain and in the time domain of many of the simple relaxation models which have been proposed. The true time domain behavior of virtually all of these relaxation models (all except the Debye model) we have had to derive for the first time. An asymptotic solution for the Cole—Cole decay was obtained by Madden and Cantwell [1967] for very long times and an asymptotic solution for the Drake model decay was obtained by Van Voorhis et al. [1973] for very short times. However, these asymptotic solutions can not be accurately applied for intermediate times, where precise observations can be made most easily, and where most of the information on the relaxation parameters is located.

Under ideal circumstances, with the transfer function analysis approach outlined in this paper, we should be able to collect rock electrical data in either the frequency domain or the time domain and then invert to identify and characterize, all of the important conduction processes occuring in the rock. In practice, of course, this sort of ideal interpretation is almost never achievable. We are always faced with the following difficulties: 1) measurements of finite accuracy over a limited frequency band or time range, containing transient effects produced by time or frequency waveforms of finite length, 2) the basic non-uniqueness produced by the infinite number of possible equivalent circuits for the rock conduction, 3) the inherent ambiguity between metallic electrode polarization and membrane polarization and even between complex resistivity and complex permittivity (all our IP behavior, for example, could be considered due to an abnormally large, complex dielectric constant), and 4) the spectral smearing due to ranges of grain size, variable pore width and interpore connections, etc.

In spite of these difficulties we believe that we can use the data analysis and inversion techniques outlined herein to 1) discern the most reasonable model out of several models proposed for a particular data set, and thereby perform limited identification of physical processes, 2) conduct quantitative evaluation of spectra so that variations in response with density, temperature, mineral composition, grain size, sulfide concentration, clay content, porosity, solution resistivity or other physical variables may be adequately characterized, and 3)
achieve accurate, and relatively easy transformation between the frequency domain and the time domain, or between the different types of measurements made in either domain.

Perhaps the most important feature of our paper, we believe, is that the theory and technique outlined herein, are relatively simple to implement. All of the forward problems which we have discussed may be programmed easily on a pocket calculator. All of the inversions of spectral data were carried out on the University of Utah Univac 1108 computer. The time required for each inversion was less than 1 second (for virtually any marginally reasonable initial guess) and the memory requirements were less than 40K. No attempt was made to optimize time or memory requirements since the cost for each inversion was always less than 10 cents.

Acknowledgements

The authors wish to acknowledge the encouragement, financial support, and equipment provided by Dr. P. H. Nelson and Dr. G. W. Hohmann of Kennecott Exploration, Incorporated, and by Mr. J. Roth of Amex Exploration, Ltd. Additional financial support was provided by National Science Foundation Grant GA-31571. Constructive discussions were held with Dr. Ralph Shuey on relaxation response and with Mr. Wesley Wilson on transformation from the frequency domain to the time domain. The spectral data from drill cores obtained at Roosevelt Hot Springs were provided by Mr. Alan Tripp. The interpretation of the spectral data from the pyrite electrodes was carried out in conjunction with Mr. James D. Klein, and the measurement of the Scott-West rocks was assisted by Mr. Grant DeWitt. Mr. J. Wong was very helpful in pointing out an error in our processing of Grisseman's data. Finally, Dr. S. H. Ward was responsible for the original direction and ultimate supervision of this research as well as creating the rock properties laboratory and academic environment which allowed the work to proceed.

REFERENCES

Collett L. S., 1959: Laboratory investigation of overvoltage, in Overvoltage research and geophysical applications. New York, Pergamon Press
Grisseman C., 1971: Examination of the frequency dependent conductivity of ore-containing rock on artificial models. Scientific Report No. 2, Electronics Laboratory, University of Innsbruck, Austria


Nelson P. H., 1975: Personal communication

Nelson P. H., Van Vorhis G. D., 1977: Discussion on “Recent advances and applications in complex resistivity measurements”. Geophysics, 42, pp. 120–121


Scott W. J., West G. F., 1969: Induced polarization of synthetic, high-resistivity rocks containing disseminated sulfides. Geophysics, 34, pp. 87–100


Tripp A., 1976: Personal communication


Interpretation of complex resistivity and dielectric data

KOMPLEX ELLENÁLLÁS- ÉS DIELEKTROMOS ADATOK ÉRTELMEZÉSE
II. RÉSZ

W. H. PELTON, W. R. SILL, В. Д. SMITH

A vezetési áram és az eltolási áram fogalmát felhasználva arra törekedtünk, hogy világosan megkülönböztessük a komplex ellenállás jelenséget és a dielektromos viselkedést. Ennek eredményeként lehetővé vált, hogy a ridge regression inverzió felhasználásával egyidejűleg megkapjuk azokat a paramétereket, amelyek az egyes jelenségeknek a megfigyelte spektrumhoz való hozzájárulását leírják. Hasonló módon más folyamatok, mint a membrán polarizáció, kettősáradó kapacitás és induktív elektromágneses csatolás azonosítása is — korlátozott mértékben — elvégezhető.

A közettulajdonságok vizsgálatában azonban az inverzióval az a legfontosabb felhasználása, hogy a spektrális görbéket néhány számú alakítja át, így a spektrum olyan pontos fizikai változók függvényében való változásait, mint a hőmérséklet, ásványos összetétel, koncentráció és szemcseméret, pontosan le lehet írni, vagy jellemezni.

Az egyszerű relaxációs modellek mindhárom tartományban (frekvencia, idő és eloszlás függvény) való vizsgálatának az is eredménye, hogy az egyik tartományban végzett szélessávú mérést át lehet számitani egy másik tartományba, vagy ugyanabban a tartományban végzett másfajta mérésre, például az amplitúdót fázissá. Ez úgy történik, hogy a ridge regression inverzió felhasználásával egyszerű relaxációs modellek összegét illesztjük a mért adatokhoz. A meghatározott paraméterek és a modell minden más tartományban ismert analitikus képlete aztán felhasználható a megfigyelt adatok bármely kivánt módon való ábrázolására.

ИНТЕРПРЕТАЦИЯ ДАННЫХ О КОМПЛЕКСНЫХ СПЕКТРАХ СОПРОТИВЛЕНИЯ И ДИЭЛЕКТРИЧЕСКИХ СПЕКТРАХ
Часть II

В. Г. ПЕЛТОН, В. Р. СИЛ, Б. Д. СМИТ

Были сделаны на основании понятии тока проводимости и тока смещения явно различить явление комплексного сопротивления и диэлектрическое поведение. В результате этого стало возможным при применении инверсии по регрессии ридж одновременно получить параметры, которые описывают вклад отдельных явлений в наблюденный спектр. Подобным образом в ограниченной мере можно распознавать такие другие процессы, как мембранную поляризацию емкость двойного пласта и индуктивную электромагнитную связь.

При изучении свойств горных пород, однако, важнейшим видом применения инверсии является преобразование спектральных кривых в некоторые цифры, таким образом изменение спектра в зависимости от важных физических переменных, в том числе температуры, минерального состава, концентрации и размера зерен, можно точно описать или характеризовать.

Результатом изучения простых релаксационных моделей во всех трех областях (функций частоты, времени и распределения) является между прочим и то, что проведенное в одном из областей широкополосное измерение можно перечислить в другой область или в другой бид измерения, проведенного в одном и том же области, напр., амплитуду в фазу. Это производится так, что с помощью инверсии по регрессии ридж сумма простых релаксационных моделей согласуется с измеренными данными. Определенные параметры и аналитическая формула модели, известная по всем другим диапазонам, могут быть применены для представления наблюдений по любому желаемому способу.